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Calculation of acoustic radiation of an open turbulent flame with a transient boundary element method

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Stütz, Michael [1]; Ochmann, Martin [2]
University of Applied Sciences Berlin, Luxemburger Str. 10, 13353 Berlin, Germany;
[1]stuetz@tfh-berlin.de [2] ochmann@tfh-berlin.de

ABSTRACT
Current BEM mostly calculates radiation for a single frequency. Transient BEM provides results for a range of frequencies directly, but has received little attention due to instability and high computational cost. Transient acoustic radiation from an open turbulent flame is calculated by using the so-called Kirchhoff or time-domain boundary integral equations. The calculation is based on velocity distribution over a cylindrical control surfaces computed with a Large Eddy Simulation (LES). The results of the calculations are presented and compared with the measured sound power of the flame and with frequency domain BEM calculations.

INTRODUCTION
The boundary element method (BEM) is a widely used numerical tool. For transient problems, the BEM is mostly formulated in frequency or Laplace domain followed by an inverse transformation. Mansur and Brebbia [1],[2] developed one of the first boundary element formulations in the time domain for the scalar wave equation and for elastodynamics with zero initial conditions. The extension of this formulation to non-zero initial conditions was presented by Antes [3]. Later Jäger [4] and Antes and Baaran [5] extended the time domain formulations to analyze 3D noise radiation caused by moving sources.
Aim of this work is to compare time domain BEM (TD-BEM) and frequency domain BEM (FD-BEM) calculations by considering an example of acoustic radiation of an open turbulent flame.

NUMERICAL MODEL
The well known time-domain boundary integral equation can be written as

$$4\pi E(x)p(x,t)=\int_\Gamma \frac{1}{r}q(y,t-r/c)d\Gamma_y+\int_\Gamma \frac{\partial r}{\partial n_y}\left[\frac{1}{r^2}p(y,t-r/c)+\frac{1}{rc}\dot{p}(y,t-r/c)\right]d\Gamma_y. \quad \text{Eq.(1)}$$

Here *p(x,t)* is the sound pressure, *r* is the distance between the points *x* and *y*, *c* is the speed of sound, Γ is the boundary surface and *q(x,t)* is the flux, following the relationship derived from Euler's equation

$$q(x,t)=-\rho_0\frac{\partial v_n(x,t)}{\partial t}, \quad \text{Eq.(2)}$$

with v_n being the normal velocity. *E(x)* is defined as follow

$$E(x)=\begin{cases}1 & ,x\in\Omega\setminus\Gamma\\ \dfrac{\alpha}{4\pi} & ,x\in\Gamma\\ 0 & ,x\notin\Omega\end{cases}, \quad \text{Eq.(3)}$$

with α being the inner solid angle and Ω the domain. A detailed derivation of the above boundary integral equation can be found for example in Araujo et al. [6] and Meise [7].

To obtain the time-discretized version of Eq.(1) a backward difference scheme for the derivative with respect to time and a linear time interpolation function for *p* are used. Time itself is discretized in equidistant time steps $t_i = i\Delta t \quad (i = 1, 2, ...)$, and so we get for the retarded time $t_{ri} = i\Delta t - r/c$. The geometry of Γ is divided into *N* planar elements $\Gamma_n \quad (n = 1, 2, ..N)$ with a uniform pressure distribution p_m^n.

$$
\begin{aligned}
q(y, t_{ri}) &= \sum_{m=1}^{i} q_m(y)\psi_m(t_{ri}), \\
p(y, t_{ri}) &= \sum_{m=1}^{i} \left(\frac{t_m - t_{ri}}{\Delta t} p_{m-1}(y) - \frac{t_{ri} - t_{m-1}}{\Delta t} p_m(y) \right) \psi_m(t_{ri}), \\
\psi_m(t_{ri}) &= \begin{cases} 1 & ,\text{if} \quad t_{ri} \in [t_{m-1}, t_m) \\ 0 & ,\text{if} \quad t_{ri} \notin [t_{m-1}, t_m) \end{cases}.
\end{aligned}
$$

Eq.(4)

For causality reasons $p(y, t_{ri}) = 0 \quad \text{for} \quad i\Delta t - r/c < 0$.

$$
4\pi E(x)p_i(x) = \sum_{n=1}^{N} \sum_{m=1}^{i} \left[\int_{\Gamma_n} \frac{1}{r} q_m^n \psi_m \, d\Gamma_n + \int_{\Gamma_n} \frac{\partial r}{\partial n_y} \frac{1}{r^2} \left[(i - m + 1)p_m^n - (i - m)p_{m-1}^n \right] \psi_m d\Gamma_n \right]
$$

Eq.(5)

For each time step the integration must only be performed over the intersection of the boundary elements Γ_n with a spherical shell with inner and outer radius $(i - m)c\Delta t$ and $(i - m + 1)c\Delta t$ respectively. Hence the resulting matrices are very sparse.

The integrals are evaluated using Gaussian quadratur. To derive a system of algebraic equations, the collocation method with respect to space and time is used. After solving the system of equations with an iterative Least-Square-Solver [8], the pressure and normal velocity on the control surface (Kirchhoff surface) are known and the sound field in the ambient medium can be calculated directly from these variables. The necessary data at the Kirchhoff surface surrounding the combustion zone was delivered by a Large Eddy Simulation (LES).

Numerical validation tests

A common problem of the TD-BEM reported in the literature is the occurrence of spurious instabilities. In case of instability the results start to oscillate at high frequency with exponentially rising amplitude. The instable behaviour depends on the ratio of time step size Δt and element size *h*. Defining $\beta = c\Delta t / h$ one observes that one gets accurate and stable results, only if β lies in a certain interval.

The stability and the accuracy of the method are investigated using a simple example. The sound pressure emitted by a harmonic pulsating sphere of radius r=1m consisting of 296 elements is calculated for different time steps [Fig.1]. A comparison with the analytic solution shows that in a range of 0.7<β<1.5 around the optimal value of $\beta = 1$ one receives correct results. Outside this range results are still stable but inaccurate. For small β this inaccuracy may result from an insufficient Gauss Order of the numerical Integration, because for each time step only a small fraction of an element has to be considered. For large β the time step is getting too large to properly resolve the amplitude of the oscillation and very large time steps can not resolve the oscillation at all.

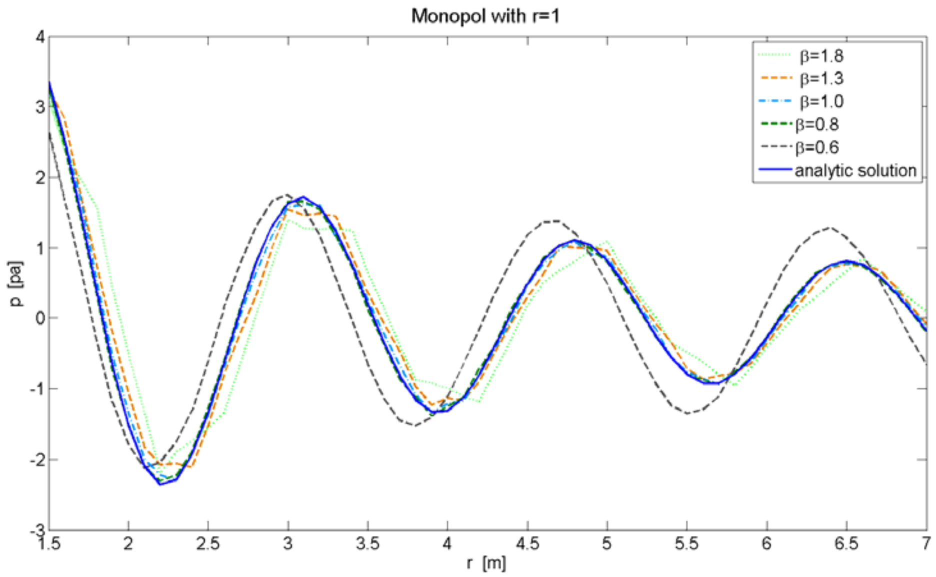

Figure 1- numerical accuracy for different values of β sphere

FLAME MODEL

In the research project "Combustion Noise", supported by the German Research Foundation (DFG) [9], a turbulent H2/N2 jet flame, referred to as "H3" flame, was simulated with an incompressible Large Eddy Simulation (LES). The values of the velocity field at cylindrical surfaces surrounding the flames delivered by the LES were used as input data to determine the radiated sound. Measurements on the flame have shown that this LES code was able to properly simulate their flow and chemical mean values [10].

Based on the velocity data prescribed on a control surface, which encloses the flame source region, the boundary element method is able to determine the acoustical field outside the control surface by evaluating the surface data. For the FD-BEM the temporal signal from the LES calculation has to be converted to the frequency domain by Fast Fourier Transform (FFT).

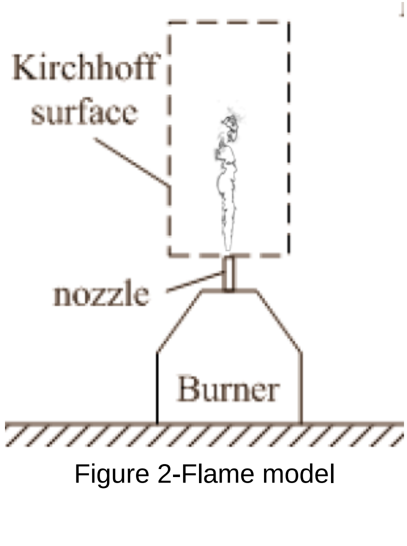

Figure 2-Flame model

REVIEW OF THE RESULTS

For comparison of the TD-BEM results with frequency domain data the transient results are transferred into frequency domain via FFT. The cylindrical control surface is divided into 1376 Elements of different sizes. Hence for the chosen time step of 0.1 ms β lies between 1.9 and 2.6 respective the element size. This is far from being the optimum value but there were no other data available from the LES calculations. Even for this suboptimal time step/mesh size we get stable results. Figure 3 shows the sound power density calculated by TD-BEM and FD-BEM compared to measurement data of a H3-Flame. Both simulation results for the H3-Flame differ notably from the measured spectra. The sound power is strongly overestimated in the considered frequency range. A turbulent velocity distribution through closed control surface at the downstream end may disturb the acoustic simulations. A further discussion of the differences between simulations and measurement can be found in Flemming et al. [10] and Piscoya et al. [11].

The TD-BEM and FD-BEM simulations lead to nearly the same results. Only in the lower frequency range up to 1 kHz the sound power density calculated by the TD-BEM is strongly overestimated. Above 1 kHz there is a relative good agreement in amplitude and frequency characteristics. In radial direction of the cylindrical control surface the directivity pattern is nearly omni-directional and is not shown here. Figure 4 shows the directivity pattern in the plane through to the radius and axis of the cylinder with the upper cap of the cylinder at 90° [see Figure 2]. For high frequencies the flame is radiating sound particularly in direction of the upper cap. The radiation pattern of TD-BEM and FD-BEM show a good agreement.

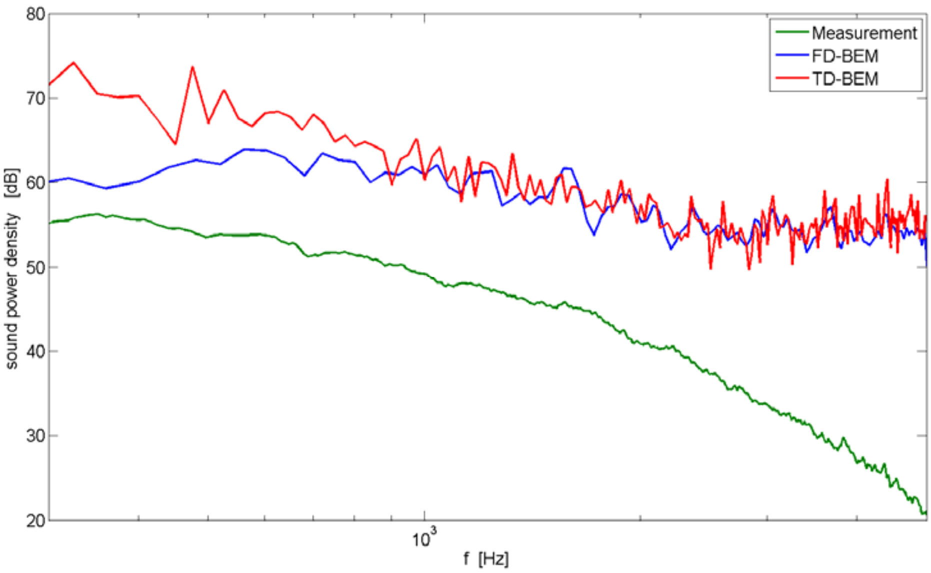

Figure 3- sound power density level of time and frequency domain calculations and measured spectrum

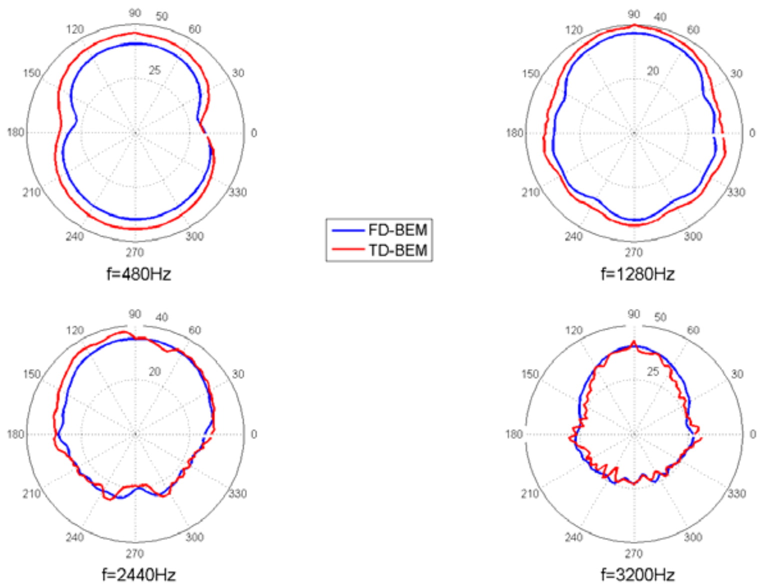

Figure 4- directivity pattern of the sound pressure level in dB for time and frequency domain calculation

CONCLUSIONS

The comparison of the results shows that the radiated sound power of the considered flame is generally overestimated. Further improvements are required in order to allow a better prediction of the combustion noise. Comparing TD-BEM and FD-BEM calculations results agree relatively well. The difference in the low frequency domain of these two approaches can not be explained so far and has to be investigated in more detail. Numerical costs of the TD-BEM are quite high. But if only the boundary values change, e.g. for a different flame, and the mesh and time step sizes would stay the same, the method would be much faster because the matrices arising from Equation 5 would not change and could be reused.

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