The Sixteenth International Congress on Sound and Vibration
Kraków, 5-9 July 2009

NUMERICAL PREDICTION OF SOUND TRANSMISSION THROUGH SHELLS AND PLATES

Rafael Piscoya and Martin Ochmann

Beuth Hochschule für Technik - University of Applied Sciences Berlin, Luxemburger Str. 10, 13353 Berlin, Germany
e-mail: piscoya@beuth-hochschule.de

In the present paper, the scattered and transmitted sound fields caused by the presence of simple structures like a spherical shell filled with fluid and a baffled plate are determined. For the spherical shell, two solutions are presented and compared for certain shell configurations. The first method solves the differential equations for a thin spherical shell, whereas the second approach considers the shell as an elastic waveguide with a finite thickness. These results are also compared with a boundary element calculation. For the baffled plate, the sound field at both sides is obtained through a Rayleigh integral. This integral is obtained from a direct boundary integral formulation, where the normal derivative of the Green's function vanishes due to the presence of the infinite baffle. The normal velocity of the plate is modelled as a sum of vibration modes of the plate obtained by a finite element simulation.

1. Introduction

The scattering of sound waves is used for detection/evaluation/classification of submerged objects in underwater acoustics. The transmission of sound through plate-like structures is determined in building acoustics to characterize sound insulation between adjacent rooms. Both situations involve a fluid-structure interaction, i.e. sound radiation and structure oscillations are coupled. When dealing with complex structures, which is mostly the case for practical applications, numerical methods have to be used. In underwater acoustics, the scattered waves can be determined using the Boundary Element Method since the medium is unbounded. For the sound transmission in buildings, the BEM is also very useful, although for the interior problem other methods like the FEM or modal decomposition are also available. For the case of a baffled plate, which will be handled here, the Helmholtz Integral Equation which is the basis of the BEM, turns into the Rayleigh integral.

Submerged objects can have different structures, here they are assumed to be made of an elastic shell filled with fluid. The plates are considered to have a small thickness compared to its length and width, therefore only the displacement normal to its surface is considered. In the present work, the approach for the numerical solution of both type of problems is presented. For the validation of the procedure, two simple structures are considered, a spherical shell for the scattering problem and a baffled plate for the transmission problem.

2. Fluid-Structure Coupling

We consider a linear elastic structure that separates two volumes (1 and 2) filled with fluid. The structure can be induced to oscillate by an external force *F*. At the same time, the oscillation of the structure affects the load that is acting on the surface. In this coupled problem, the unknowns are the displacement of the structure u and the pressures at both of its sides, p_1 and p_2

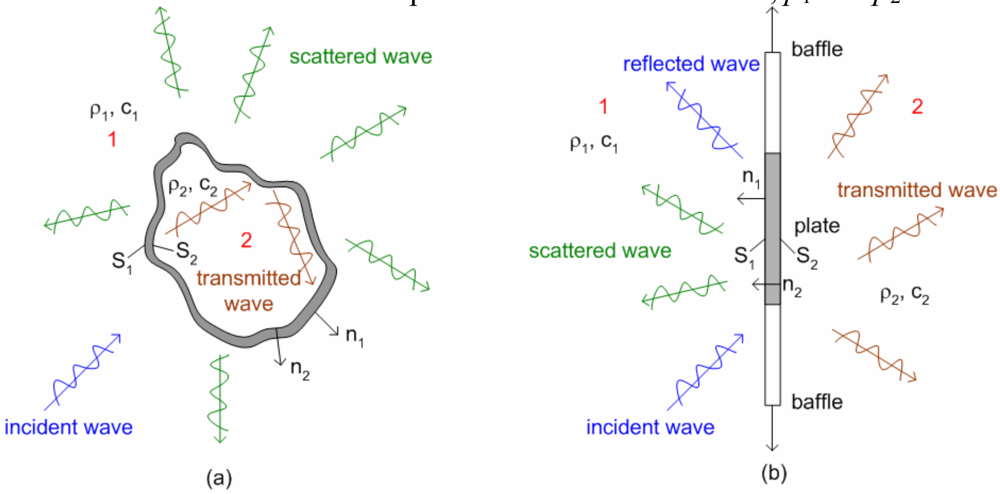

Figure 1. Illustration of the problems; a) scattering; b) transmission.

The equations of motion of the structure subject to the boundary conditions are obtained using a variational formulation and the extremalization of the Hamiltonian of the structure.

$$\delta H = 0 \quad , \quad H = \int_{t_0}^{t_1} \left(T - V - V_b + W\right) dt \,. \tag{1}$$

where T is the kinetic energy, V and V_b are the potential energies of the structure and due to the boundary conditions respectively and W is the work associated to the external forces and loads.

For the submerged object, the radiated sound pressures satisfy the Helmholtz Integral Equations (HIE):

$$\begin{aligned} C_1 p_1 &= \int_{S_1} \left(p_1^S \frac{\partial g_1}{\partial n_1} - \rho_1 \omega^2 u_{n1}^S g_1 \right) dS + p_{inc} && \text{in } 1 \\ C_2 p_2 &= -\int_{S_2} \left(p_2^S \frac{\partial g_2}{\partial n_2} - \rho_2 \omega^2 u_{n2}^S g_2 \right) dS && \text{in } 2 \end{aligned} \tag{2.a}$$

with $$g_1 = \frac{e^{-jk_1|\vec{x}-\vec{y}|}}{4\pi|\vec{x}-\vec{y}|} \;,\; g_2 = \frac{e^{-jk_2|\vec{x}-\vec{y}|}}{4\pi|\vec{x}-\vec{y}|} \;,\; C_1 = \begin{cases} 1 & \text{outside } S_1 \\ 0.5 & \text{on } S_1 \\ 0 & \text{inside } S_1 \end{cases} \;,\; C_2 = \begin{cases} 1 & \text{inside } S_2 \\ 0.5 & \text{on } S_2 \\ 0 & \text{outside } S_2 \end{cases} \,.$$

For the transmission through the baffled plate, the HIE turns into a Rayleigh integral:

$$\begin{aligned} p_1 &= p_{inc} + p_{ref} - \rho_1 \omega^2 \int_{S_1} u_{n1}^S g_{H,1} dS && \text{in } 1 \\ p_2 &= \rho_2 \omega^2 \int_{S_2} u_{n2}^S g_{H,2} dS && \text{in } 2 \end{aligned} \tag{2.b}$$

where p_{ref} is the sound wave reflected at the baffle without the plate and

$$g_{H,1} = \frac{e^{-jk_1|\vec{x}-\vec{y}|}}{2\pi|\vec{x}-\vec{y}|} \;,\; g_{H,2} = \frac{e^{-jk_2|\vec{x}-\vec{y}|}}{2\pi|\vec{x}-\vec{y}|} \;,\; \frac{\partial g_{H,1}}{\partial n} = \frac{\partial g_{H,2}}{\partial n} = 0 \,. \tag{3}$$

The solution of the coupled problem of Eqs. (1) and (2.a) or (2.b) is obtained at a finite number of points (nodes) inside the structure and at its surface. The discretized form of Eq. (1), written in matrix form is given by:

$$\left(K - \omega^2 M\right) u = F + F^m \,. \tag{3}$$

where M is the mass matrix, K is the stiffness matrix, F and F^m are the vectors of external forces and loads respectively and u is the vector of structural nodal displacement.

Discretizing Eqs. (2.a) and (2.b) at the surfaces S_1 and S_2, we obtain the matrix expressions:

$$\begin{aligned} (0.5I - H_1)p_1^S &= p_{inc}^S - \rho_1\omega^2 G_1 u_{n1}^S && \text{in } 1 \\ (0.5I + H_2)p_2^S &= \rho_2\omega^2 G_2 u_{n2}^S && \text{in } 2 \end{aligned} \tag{4.a}$$

$$\begin{aligned} p_1^S &= p_{inc}^S + p_{ref}^S - \rho_1\omega^2 G_1 u_{n1}^S && \text{in } 1 \\ p_2^S &= \rho_2\omega^2 G_2 u_{n2}^S && \text{in } 2 \end{aligned} \tag{4.b}$$

Combining Eqs. (3) and (4) and assuming that there are no external forces (F=0), the coupled system of equations for u, p_1^S and p_2^S can be obtained:

$$\begin{bmatrix} K - \omega^2 M & A_1 & -A_2 \\ \rho_1\omega^2 G_1 R_1 & 0.5I - H_1 & 0 \\ -\rho_2\omega^2 G_2 R_2 & 0 & 0.5I + H_2 \end{bmatrix} \begin{bmatrix} u \\ p_1^S \\ p_2^S \end{bmatrix} = \begin{bmatrix} 0 \\ p_{inc}^S \\ 0 \end{bmatrix}. \tag{5.a}$$

$$\begin{bmatrix} K - \omega^2 M & A_1 & -A_2 \\ \rho_1\omega^2 G_{H,1} R_1 & 1 & 0 \\ -\rho_2\omega^2 G_{H,2} R_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ p_1^S \\ p_2^S \end{bmatrix} = \begin{bmatrix} 0 \\ p_{inc}^S + p_{ref}^S \\ 0 \end{bmatrix}. \tag{5.b}$$

where A_1 and A_2 are matrices containing the surface associated to each node at the surface of the structure and R_1 and R_2 are matrices that have zeros for nodes that are not located on the surface and values different from zero for nodes that are located on the surface.

If the elastic structure is very thin, one can use shell elements with the appropriate thickness instead of volume elements. In that case, S_1=S_2, A_1=A_2 and R_1=R_2 and assuming there are no internal substructures, all nodes will be at the surface.

The system of Eqs. (5) can be reduced to a system of equations for u only:

$$\left(\rho_1\omega^2 A_1 (0.5I - H_1)^{-1} G_1 R_1 + \rho_2\omega^2 A_2 (0.5I + H_2)^{-1} G_2 R_2 - \left(K - \omega^2 M\right)\right)u = A_1 (0.5I - H_1)^{-1} p_{inc}^S. \tag{6.a}$$

$$\left(\rho_1\omega^2 A_1 G_1 R_1 + \rho_2\omega^2 A_2 G_2 R_2 - \left(K - \omega^2 M\right)\right)u = A_1\left(p_{inc}^S + p_{ref}^S\right). \tag{6.b}$$

and the other variables are obtained directly from u using the relations:

$$p_1^S = (0.5I - H_1)^{-1}\left(p_{inc}^S - \rho_1\omega^2 G_1 R_1 u\right) \quad , \quad p_2^S = (0.5I + H_2)^{-1}\rho_2\omega^2 G_2 R_2 u. \tag{7.a}$$

$$p_1^S = p_{inc}^S + p_{ref}^S - \rho_1\omega^2 G_{H,1} R_1 u \quad , \quad p_2^S = \rho_2\omega^2 G_{H,2} R_2 u. \tag{7.b}$$

The system of equations in (6) can be very large if the discretization of the object is very fine, but it can be reduced if the displacement vector is expressed as a linear combination of structural modes in vacuum:

$$u(\vec{x}) = \sum_n \alpha_i \phi_i(\vec{x}) \quad \rightarrow \quad u = \phi\alpha. \tag{8}$$

The coefficients α_i are called participation factors. Normally, they decrease with the order of the modes, so the number of modes needed to achieve results within a certain error margin should be much lower that the number of nodes. Assuming there is no damping in the structure, the modes are orthogonal and satisfy the equation

$$(K - \omega_i M)\phi_i = 0 \quad \text{with} \quad \phi_j^T K \phi_i = \phi_j^T M \phi_i = 0 \;,\; i \neq j \tag{9}$$

Replacing Eq. (8) in (6) and multiplying both sides by ϕ^T we obtain

$$\left(\rho_1\omega^2 Q_1 + \rho_2\omega^2 Q_2 - \left(\tilde{K} - \omega^2 \tilde{M}\right)\right)\alpha = b, \quad \tilde{K} = \phi^T K \phi \quad , \quad \tilde{M} = \phi^T M \phi \tag{10}$$

$$Q_1 = \phi^T A_1 (0.5I - H_1)^{-1} G_1 R_1 \phi \;,\; \mathrm{Q}_2 = \phi^T A_2 (0.5I + H_2)^{-1} G_2 R_2 \phi \;,\; b = \phi^T A_1 (0.5I - H_1)^{-1} p_{inc}^S. \tag{11.a}$$

$$Q_1 = \phi^T A_1 G_{H,1} R_1 \phi \quad , \quad \mathrm{Q}_2 = \phi^T A_2 G_2 R_2 \phi \quad , \quad b = \phi^T A_1 \left(p_{inc}^S + p_{ref}^S\right). \tag{11.b}$$

The matrix to be inverted in Eq. (10) is a $n \times n$ square matrix with $n<N$, n=number of modes, N=number of nodes. Because of the orthogonality of the eigenvectors and the symmetry of K and M, the matrices $\widetilde{K}$ and $\widetilde{M}$ are diagonal.

3. Test examples

The procedure described in section 2 was tested using two benchmarks problems, the scattering of a plane wave by a spherical thin shell and the sound transmission through a baffled simply supported rectangular plate.

3.1 Scattering by a thin spherical shell

The scattering of a thin spherical shell can be obtained analytically using two different approaches. The first one is to solve the equation of motion for an elastic waveguide with a finite thickness and the second one is to use the Kirchhoff-Love theory for a spherical shell.

3.1.1 Elastic Theory

A spherical shell of radii a and b ($b>a$) is considered. The mechanical vibration of the elastic structure is governed by the equation

$$\frac{\partial^2 \vec{u}}{\partial t^2} = \alpha^2 \nabla\left(\nabla \cdot \vec{u}\right) - \beta^2 \nabla \times \left(\nabla \times \vec{u}\right) + \rho \vec{f} \tag{12}$$

where $\alpha = \sqrt{(\lambda + 2\mu)/\rho}$ and $\beta = \sqrt{\mu/\rho}$ are the P- and S-wave velocities, λ and μ are the Lamé elastic parameters and $\vec{f}$ f is the vector of external forces. The stress tensor τ_{ij} is expressed in terms of the strain tensor ε_{ij}: $\tau_{ij} = \lambda \nabla \cdot \vec{u} \delta_{ij} + 2\mu\varepsilon_{ij} \quad , \quad \varepsilon_{ij} = 0.5\left(\partial u_i / \partial x_j - \partial u_j / \partial x_i\right)$.

If external forces and torsional vibrations are not considered and the displacement vector is written as $\vec{u} = \nabla F + \nabla \times \left(\nabla \times (\vec{r} S)\right)$, two uncoupled wave equations for the scalars F and S can be obtained. These scalars can be then written as

$$F = \sum_n \left(A_n j_n(k_\alpha r) + C_n y_n(k_\alpha r)\right) P_n(\cos\theta) \quad , \quad k_\alpha = \omega / \alpha \tag{13.a}$$

$$S = \sum_n \left(B_n j_n(k_\beta r) + E_n y_n(k_\beta r)\right) P_n(\cos\theta) \quad , \quad k_\beta = \omega / \beta \tag{13.b}$$

The scattered and transmitted fields can also be expanded in spherical functions

$$p_{sc} = \sum_n D_n h_n^{(2)}(kr) P_n(\cos\theta) \quad , \quad k = \omega / c \tag{14.a}$$

$$p_f = \sum_n T_n j_n(k_f r) P_n(\cos\theta) \quad , \quad k_f = \omega / c_f \tag{14.b}$$

The coefficients A_n, B_n, C_n, D_n, E_n and T_n can be determined using the boundary conditions, which require continuity of the normal displacement and the stress acting on the shell surface.

3.1.2 Kirchhoff-Love Theory

A spherical shell of radius a is considered. If the shell is very thin, i.e. its thickness h is small compared with its other dimensions and compared with its principal radius of curvature, following assumptions are made: a) straight lines that are normal to the middle surface prior to deformation remain straight and normal to the middle surface during deformation, and experience no change in length; b) the direct stress acting in the direction normal to the shell middle surface is negligible.

If no torsional vibrations are considered ($u_\phi = 0$) and the external loads act normal to the surface, the displacement of the middle surface is given by the following set of equations [1]

$$L_{11}u_\theta + L_{13}u_r = \frac{\rho h}{Y}\frac{\partial^2 u_\theta}{\partial t^2} \quad , \quad Y = \frac{Eh}{1\text{-}\nu^2} \tag{15.a}$$

$$L_{31}u_\theta + L_{33}u_r = \frac{1}{Y}\left(-\rho h\frac{\partial^2 u_r}{\partial t^2} + p_{ext}\right) \tag{15.b}$$

$$L_{11} = \frac{1+\beta^2}{a^2}\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} - \nu - \cot^2\theta\right) \tag{15.c}$$

$$L_{13} = \frac{-\beta^2}{a^2}\left(\frac{\partial^3}{\partial\theta^3} + \cot\theta\frac{\partial^2}{\partial\theta^2} - \left(\nu + \cot^2\theta\right)\frac{\partial}{\partial\theta}\right) + \frac{1+\nu}{a^2}\frac{\partial}{\partial\theta} \tag{15.d}$$

$$L_{31} = \frac{-\beta^2}{a^2}\left(\frac{\partial^3}{\partial\theta^3} + 2\cot\theta\frac{\partial^2}{\partial\theta^2} - \left(1+\nu+\cot^2\theta\right)\frac{\partial}{\partial\theta} + \cot\theta\left(1-\nu+\frac{1}{\sin^2\theta}\right)\right) + \frac{1+\nu}{a^2}\left(\frac{\partial}{\partial\theta} + \cot\theta\right) \tag{15.e}$$

$$L_{33} = \beta^2 a^2\left(\nabla_\theta^2\nabla_\theta^2 + \frac{1-\nu}{a^2}\nabla_\theta^2\right) + \frac{2(1+\nu)}{a^2} \quad , \quad \nabla_\theta^2 = \frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} \tag{15.f}$$

where E is the Young's modulus and ν the Poisson's ratio.

The displacement components u_r and u_θ can be expanded in spherical harmonics

$$u_r = \sum_n W_n P_n(\cos\theta) \quad , \quad u_\theta = \sum_n V_n \sin\theta\, P_n'(\cos\theta) \tag{16}$$

and the scattered and transmitted field are again given by (14). Inserting (16) into (15) together with the boundary conditions of continuity of normal displacement, the coefficients W_n, V_n, D_n and G_n can be determined.

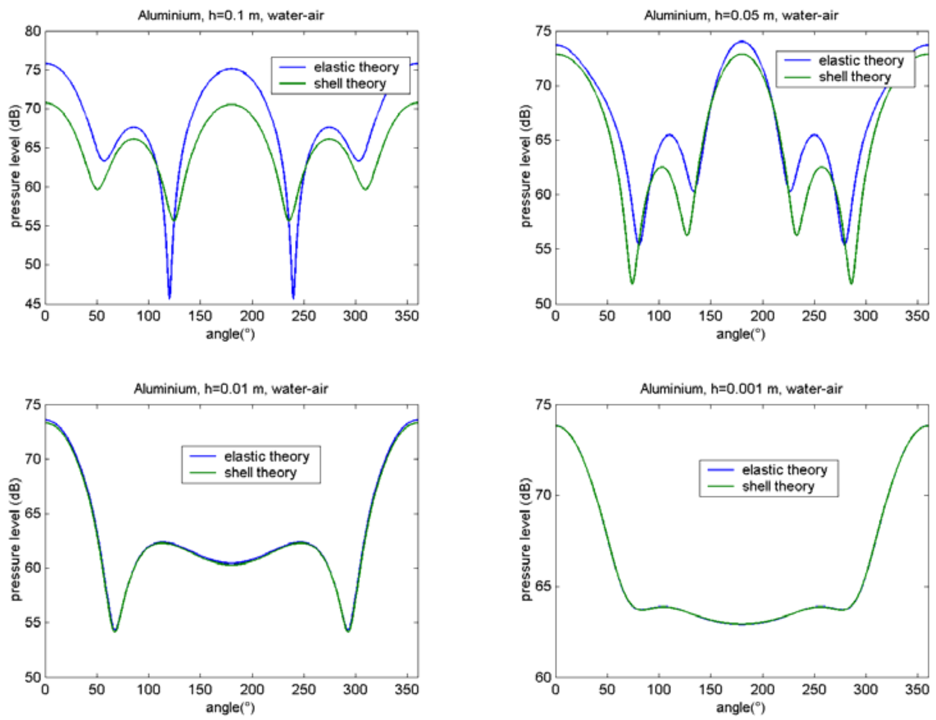

Figure 2. Scattered sound by a spherical shell. Comparison between elastic and shell theory for different thickness.

A comparison between both analytical approaches is shown in Fig. 2. The scattered sound pressure from an aluminium shell with 0.5 m radius is calculated in a circle of radius 10 m. The thicknesses was varied from 0.1 to 0.001 at a fixed frequency of 1000 Hz. Both approaches provide the same results for h<0.01, about R/h>50. The differences increase for the thicker shells.

The numerical calculation in the present work considers only the shell theory. A simulation using the elastic theory was made and presented in [2]. The sphere model with radius a=0.475 m and thickness h=0.05 m has 640 triangular shell elements. The exterior medium is water and the interior fluid is air.

The modal participation is obtained solving (10), the shell displacement is determined with (8) and the sound pressure at the shell surface using (7.a). The scattered sound pressure is computed solving the surface integral in (2.a). A comparison between analytical and numerical results are presented in Fig. (3). A very good agreement between both results is found for different values of h.

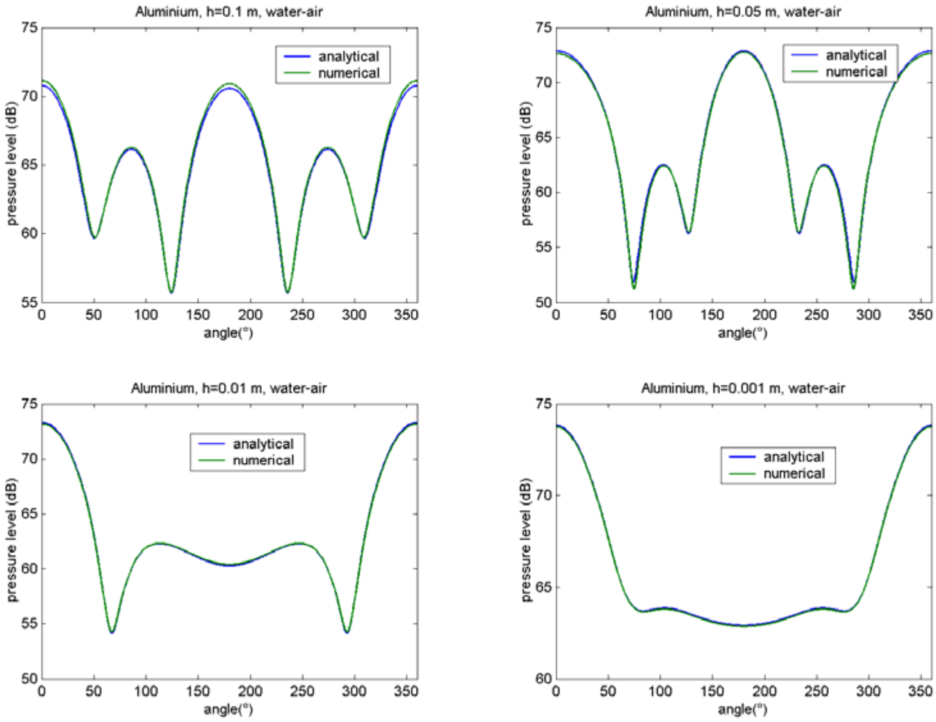

Figure 3. Scattered sound by a spherical shell. Comparison between analytical and numerical results.

3.1.3 Indirect BEM approach

For thin structures and for open structures, the indirect BEM approach is a more flexible formulation. If we consider that the same fluid is present at both sides of the structure and that the structure is thin enough to assume that the displacement of the body at its front and back sides is the same, the addition of both equations (2.a) yields to an expression of the sound pressure at every point not lying on the surface

$$p = p_{inc} + \int_S \delta p^S \frac{\partial g}{\partial n'} dS \quad , \quad \delta p^S = p_1^S - p_2^S \tag{17}$$

Here, the unknown variable is the pressure jump δp^S and not the pressure. To find the pressure jump, we perform the normal derivative of both equations (2.a) and add both equations to obtain

$$\rho\omega^2 u^S = \frac{\partial p_{inc}}{\partial n} + \int_S \delta p^S \frac{\partial^2 g}{\partial n \partial n'} dS \tag{18}$$

The integral in (18) is hypersingular but since we work with constant elements, this integral is handled using the method conceived by Osetrov and Ochmann [3]. Following this procedure, Eq. (18) is written as

$$\rho\omega^2 u^S(\vec{x}) = \frac{\partial p_{inc}(\vec{x})}{\partial n(\vec{x})} + \int_S \left(\delta p^S(\vec{y}) - \delta p^S(\vec{x})\right) \frac{\partial^2 g(\vec{x},\vec{y})}{\partial n(\vec{x})\partial n(\vec{y})} dS + \delta p^S(\vec{x}) \int_S \left(\frac{\partial^2 g(\vec{x},\vec{y})}{\partial n(\vec{x})\partial n(\vec{y})} - \frac{\partial^2 G(\vec{x},\vec{y})}{\partial n(\vec{x})\partial n(\vec{y})} \right) dS \tag{19}$$

with $G = 1/|\vec{x} - \vec{y}|$. Combining the matrix form of Eq. (19) with Eq. (3) and considering F=0, $F^m = -A\delta p^S$, we find the system of equations to be solved

$$\begin{bmatrix} K - \omega^2 M & A \\ \rho\omega^2 R & L \end{bmatrix} \begin{bmatrix} u \\ \delta p^S \end{bmatrix} = \begin{bmatrix} 0 \\ \partial p^S_{inc} / \partial n \end{bmatrix}. \tag{20}$$

where A is the matrix of the elements surface, R is the matrix identifying the nodes on the surface and L is the matrix corresponding to the surface integrals in (19).

A comparison of the results from the direct and indirect formulations with the analytical solution is shown in Fig. 4 for h=0.05 m and air outside and inside the shell. The agreement is very good for both approaches. At 1000 Hz, the error from the indirect approach is bigger than the error of the direct approach because the discretization has a stronger influence in the indirect method since the system matrix is built from the normal derivative of the HIE.

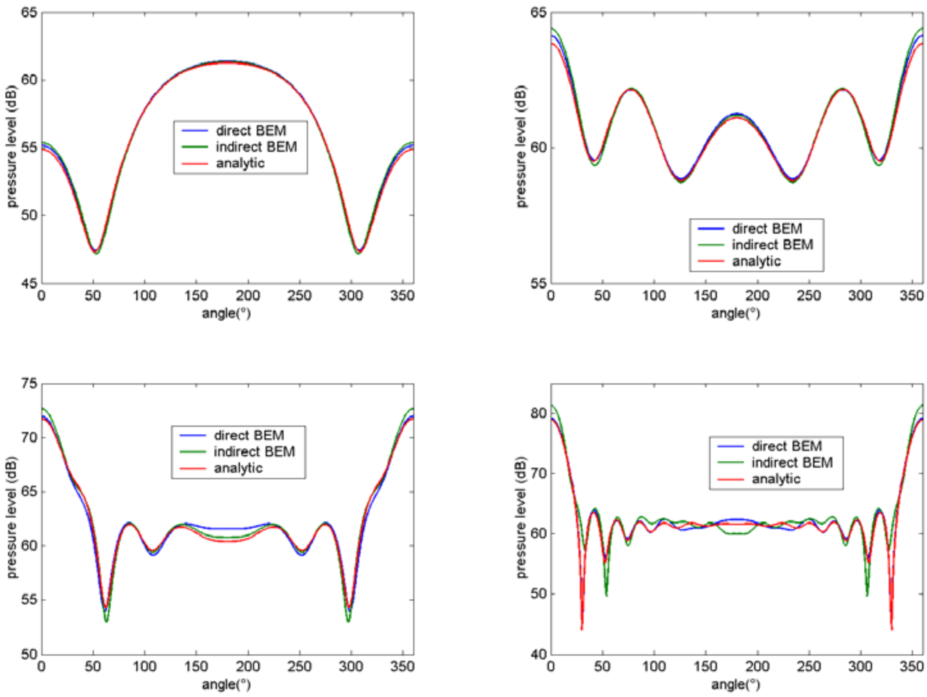

Figure 4. Scattered sound by a spherical shell. Comparison between analytical and numerical results.

3.2 Transmission through a baffled plate

The transmission efficiency (τ) of a baffled plate is given by the expression

$$\tau = \frac{W_2}{W_{inc}} \quad , \quad W_2 = \int_S 0.5\,\mathrm{Re}\left(p_2^S \left(v_{n2}^S\right)^*\right) dS \quad , \quad W_{inc} = \int_S 0.5\,\mathrm{Re}\left(p_{inc}\left(v_{n,inc}^S\right)^*\right) dS \tag{21}$$

where $v_n = j\omega u_n$ is the normal velocity. The transmission loss (TL) is defined as $TL = -10\log_{10}\tau$. The normal displacement of the plate can be obtained using Eq. (8) after solving Eq. (10). The load of the medium is contained in the matrices Q_1 and Q_2 in Eq. (10). When the media are very light compared to the plate, their effect can be neglected and the displacement will depend only on the incident wave (blocked pressure approximation).

The normal displacement depends strongly on the boundary conditions. A configuration which has a simple solution is the case of an isotropic simply supported plate of sides a and b. The normal displacement is governed by [1]

$$D\nabla^2\nabla^2 u - \rho h\omega^2 u = p_1^S - p_2^S \quad , \quad D = Eh^3/12\left(1-\nu^2\right) \tag{22}$$

and the eigenvectors and eigenfrequencies are given by

$$\phi_{nm} = \sin(n\pi/a)\sin(m\pi/b) \quad , \quad \omega_{nm} = \pi^2\left((n/a)^2 + (m/b)^2\right)\sqrt{D/\rho h} \tag{23}$$

The effect of the load of the medium in the averaged displacement of the plate and in the TL is shown in Fig. 5. Two different plates with the same dimensions were considered for the calculations. The first plate has h=0.001 and elastic properties ρ=7850 kg/m^3 and E=200GPa, the second one is thicker h=0.005 m but lighter with parameters ρ=200 kg/m^3 , E=6.4GPa. The results show that only for the second case, the load of the medium affects the oscillation of the plate.

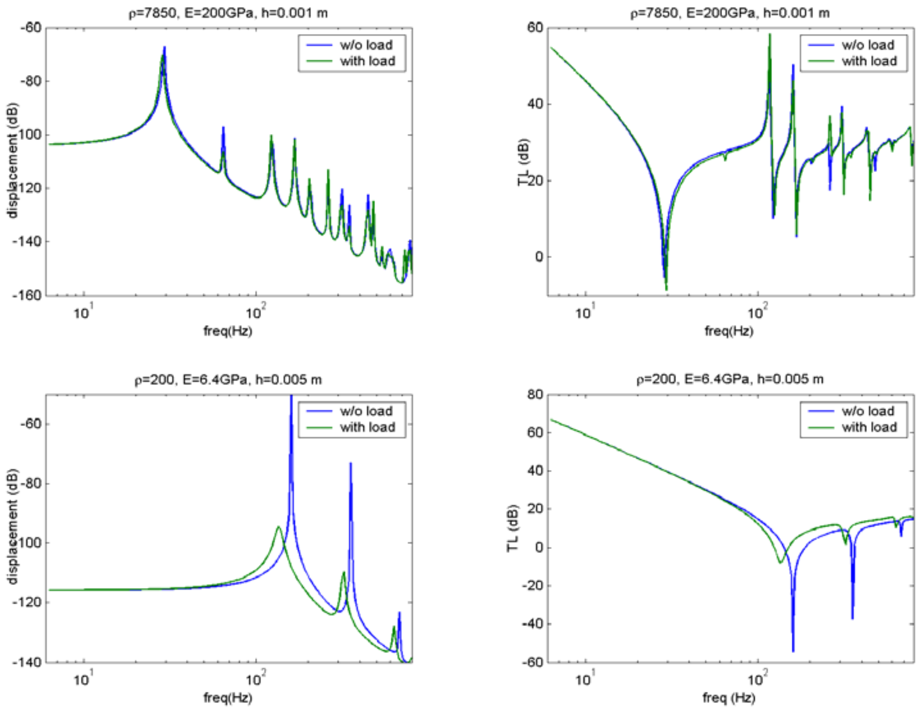

Figure 5. Comparison of displacement and TL, with and without load.

4. Summary

In the present work, a method to compute numerically the sound scattering and sound transmission considering fluid structure coupling (FSC) was presented. To determine the radiated sound, direct and indirect BEM approaches were considered. The method was tested using two simple configurations that serve as benchmark problems for the validation of any FSC solver.

REFERENCES

[1] W. Bogusz, Z Dżygadło, D. Rogula, K. Sobczyk, L. Solarz, *Vibration and Waves, Part A: Vibrations*, Elsevier, 1992

[2] R. Burgschweiger, I. Schäfer, R. Piscoya, M. Ochmann, Benchmarking for sound transmission and scattering from thin elastic structures using analytical, BE and FE coupling methods, *Proceedings NAG/DAGA*, 2009.

[3] A. Osetrov, M. Ochmann, A fast and stable numerical solution for acoustic BEM combined with the Burton and Miller methods for models consisting of constant elements, *Journal of Computational Acoustics,13(1)*, 2005.