Benchmarking for sound transmission and scattering from thin elastic structures using analytical, BE and FE coupling methods

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Introduction

The calculation of the pressure scattered from elastic structures composed of thin elastic materials is one of the main purposes for the detection of underwater objects.

For this reason, the sound pressure scattered from spherically structures ("shells") placed in and filled with fluid will be calculated using different numerical and analytical coupling methods.

We will compare and benchmark analytical solutions based on spherical wave functions with results of an in-house developed BEM-package and commercial BEM/FEM applications.

BEM coupling method

The implemented boundary element coupling method which is based on Nolte [1] can be used for all coupling types (fluid/fluid, fluid/solid und solid/solid). At this time constant elements (with a selectable Gauss point distribution) are used. An extension to linear / quadratic elements is intended.

The particular coupling type is determined by the existing geometry and the associated material parameters and is used to define the appropriate matrix coefficients.

Transition conditions exist at the structural interface points. Further on one has to take into account the incident pressure of the sound source.

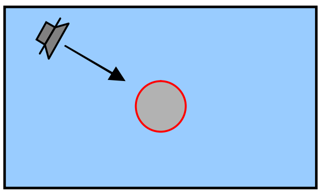

Figure 1: single coupled case for a "massive" sphere (p_{inc} affects the whole surface)

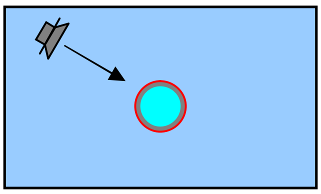

Figure 2: multiple coupled case for a spherical shell (p_{inc} affects only the outer surface of the shell)

The results for the single coupled case have been already published in [2], so we will focus here on the multi-coupled case as shown in Figure 2.

Multi-coupled problem

A spherical shell with a fixed outer diameter of 1 m and with different widths and fillings is placed in water. Using the BEM coupling method, we have three structures: the "outer space", the shell and the inner sphere (filling).

A plane wave is impinging on the shell in positive X-direction as shown in figure 3.

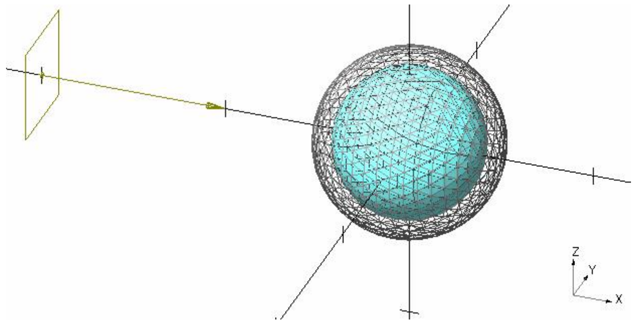

Figure 3: Spherical shell with impinging plane wave

The results were compared with results obtained by commercial FEM software (COMSOL FEMLab) and analytical solutions by Holford and Piscoya (using MatLab).

Automated Gauss point adaption

For near "opposite" elements (with $\Delta l < l_{e,min}$ as shown in figure 4) an automated adaption algorithm increases the default number of used Gauss integration points for both elements. The maximum supported number of Gauss points is 4 096 for quadrilateral and 64 for triangular elements.

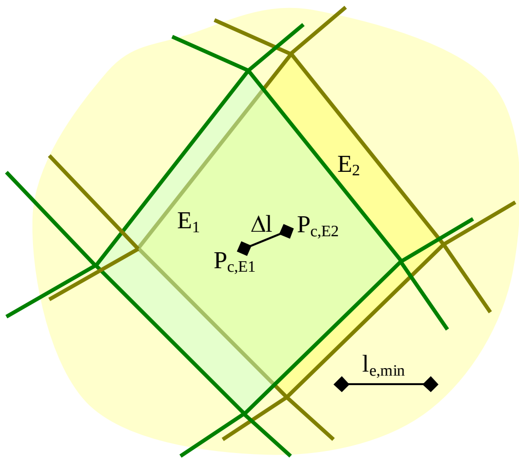

Figure 4: Opposite "near" elements

E_1: 1st element, with $P_{c,E1}$: center of E_1
E_2: 2nd "opposite" element, with $P_{c,E2}$: center of E_2

$\Delta l = |P_{c,E1} - P_{c,E2}|$

$l_{e,min}$: minimum element side length (of all mesh elements)

Test cases

We present the results for five test cases using combinations of different shell widths from 10 cm to 1 mm, two shell materials (aluminium and steel) and two filling materials (water and air).

Outer diameter for all cases:	1 m (r_A = 0.5 m)	
wavelengths (at 1 000 Hz):	λ_{water}:	1.5 m
	λ_{air}:	0.343 m
	$\lambda_{comp,aluminium}$:	6.321 m
	$\lambda_{shear,aluminium}$:	3.163 m
	$\lambda_{comp,steel}$:	5.707 m
	$\lambda_{shear,steel}$:	3.155 m

Structure-specific parameters of the shells

$Shell_{1280}$:	triangular elements:	2 014	(1 274 / 740)
	max. element size:	0.1	m
	matrix order / size:	8 056,	≈ 990 MB
	solving time:	≈ 160	s
$Shell_{5000A}$:	quad elements:	8 220	(5 046 / 3 174)
	max. element size:	0.035	m
	matrix order / size :	32 880,	≈ 16 496 MB
	solving time:	≈ 1 720	s
$Shell_{5000B}$:	quad elements:	10 092	(5 046 / 5 046)
	matrix order / size:	40 368,	≈ 24 865 MB
	solving time:	≈ 3 060	s

Calculator: Dual XEON QuadCore (total cores: 8)
running at 2.66 Ghz, 64 GB RAM

Abbreviations:

TE:	triangular elements
QE:	quad elements
GPx:	Gauss point adaption (auto/fixed)
IMKL/DS:	Intel MKL Solver (direct)

All figures are showing the absolute value of the scattered pressure on a circle r = 10 m to the center.

Test case 1: 10 cm aluminium shell, f = 1 300 Hz

outer space: water, shell: aluminium, filling: water

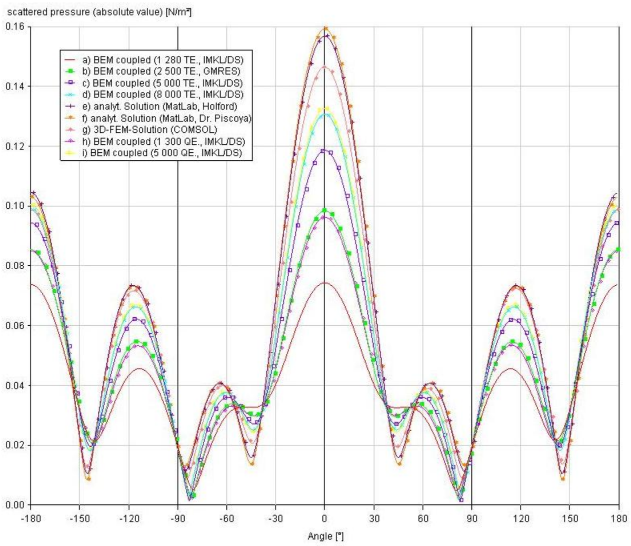

Figure 5: 10cm aluminium shell at 1 300 Hz, filled with and surrounded by water

The analytical solutions (e and f) are matching very well, the accuracy of the BEM results increases with a finer discretization and the use of quad elements (h and i).

The result of the 3D FEM calculation (g) is slightly better than the "best" BEM result.

Test case 2a: 10cm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: water

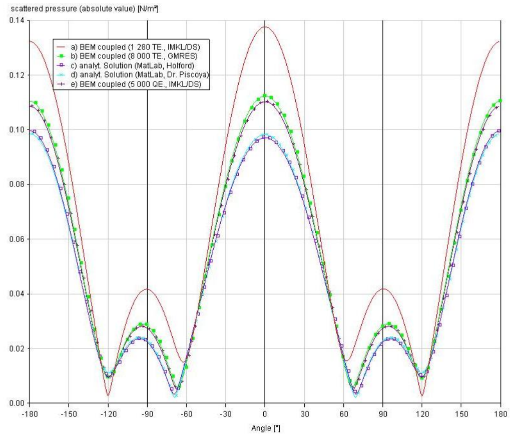

Figure 6: 10cm aluminium shell at 1 000 Hz, filled with and surrounded by water

The analytical solutions (c and d) also match very well, while the accuracy of the BEM solution still depends on the discretization.

Test case 2b: 10cm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: air

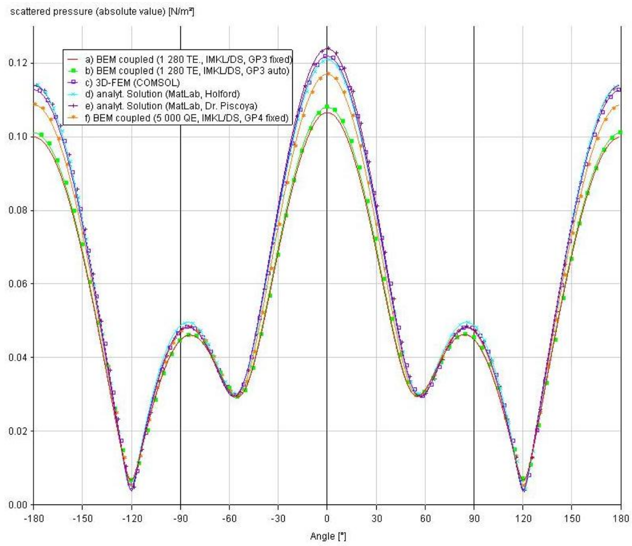

Figure 7: 10cm aluminium shell at 1 000 Hz, filled with air and surrounded by water

The analytical solutions (d and e) and the 3D FEM result (c) match very well and also the accuracy of the BEM solution gets better with the air filling.

Here one can also see that the use of the automated Gauss point algorithm (b) leads to better results than a fixed Gauss point number (a, using the same structure and discretization).

Test case 2c: 10cm steel shell, f = 1 000 Hz

outer space: water, shell: steel, filling: air

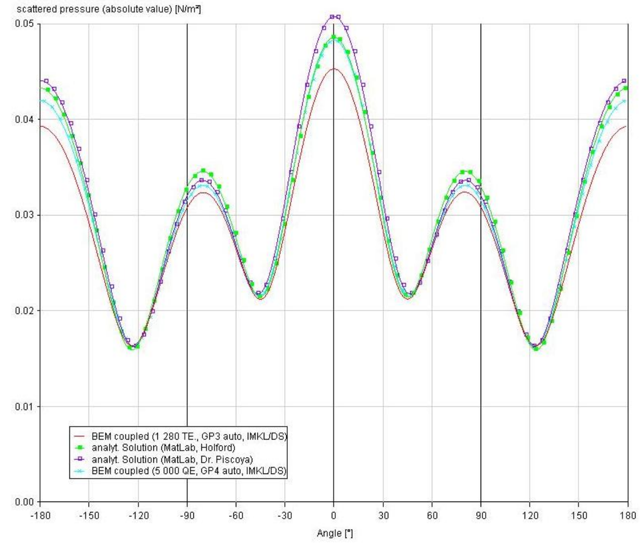

Figure 8: 10cm steel shell at 1 000 Hz, filled with air and surrounded by water

Here all results agree well.

Test case 3: 5cm steel shell, f = 1 000 Hz

outer space: water, shell: steel, filling: air

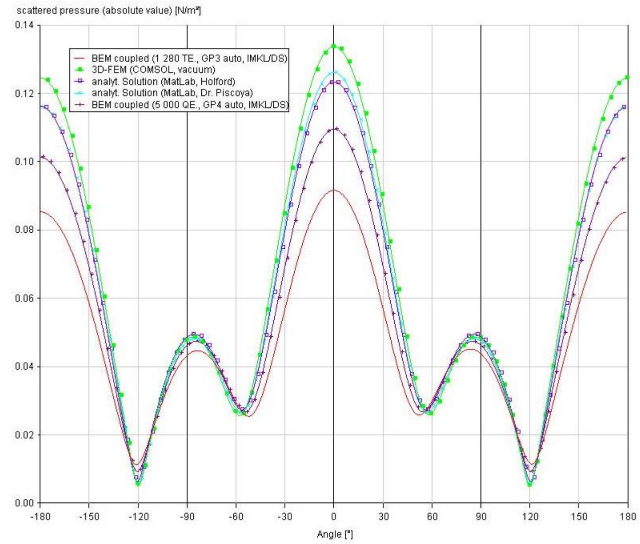

Figure 9: 5cm steel shell at 1 000 Hz, filled with air and surrounded by water

For this case, the BEM solution does not reach the same accuracy as in the other cases. The 3D FEM result is slightly higher than the analytical results due to the fact, that the filling was calculated strainless (instead of air).

Test case 4a: 1cm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: water

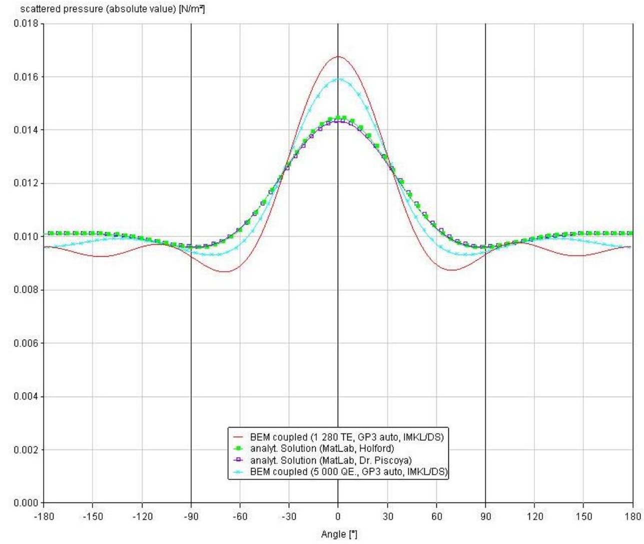

Figure 10: 1cm aluminium shell at 1 000 Hz, filled with and surrounded by water

The analytical solutions still match very well for this case, while the agreement of the BEM result with the analytical solutions decreases.

Test case 4b: 1cm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: air

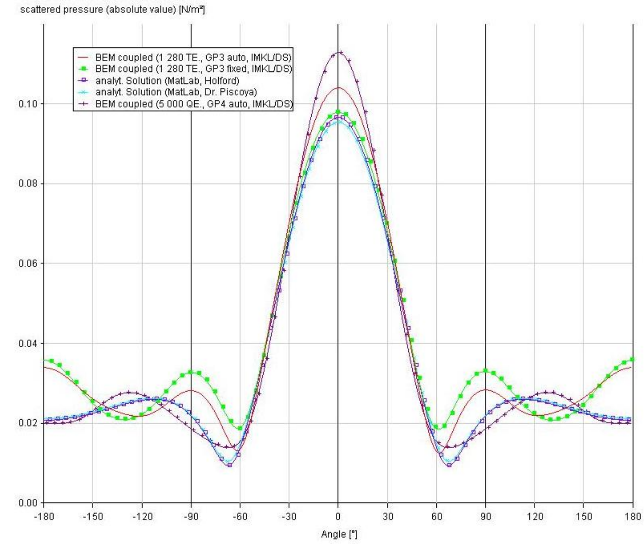

Figure 11: 1cm aluminium shell at 1 000 Hz, filled with air and surrounded by water

While the backscattered pressure (at 0°) of the BEM calculation agrees well with the analytical solutions, the characteristic of result values in the other angle ranges differ more and more.

Test case 4c: 1cm steel shell, f = 1 000 Hz

outer space: water, shell: steel, filling: air

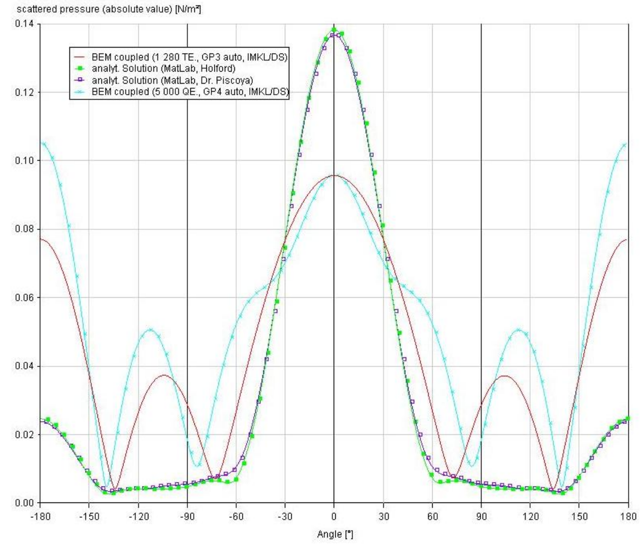

Figure 12: 1cm steel shell at 1 000 Hz, filled with air and surrounded by water

Here we have the largest differences between the analytical and the BEM solutions. At this time we do not have a 3D FEM solution to compare with due to discretization and time limits.

These differences need further investigation and may lead to modifications of the coupling method for these special “thin shell” cases.

Test case 5a: 1mm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: water

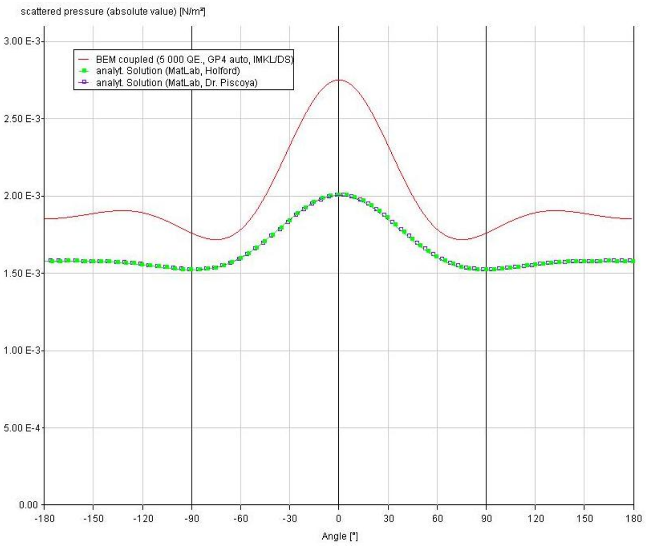

Figure 13: 1mm aluminium shell at 1 000 Hz, filled with and surrounded by water

The analytical solutions match nearly exact while the characteristic of the BEM solution still show differences.

Test case 5b: 1mm aluminium shell, f = 1 000 Hz

outer space: water, shell: aluminium, filling: air

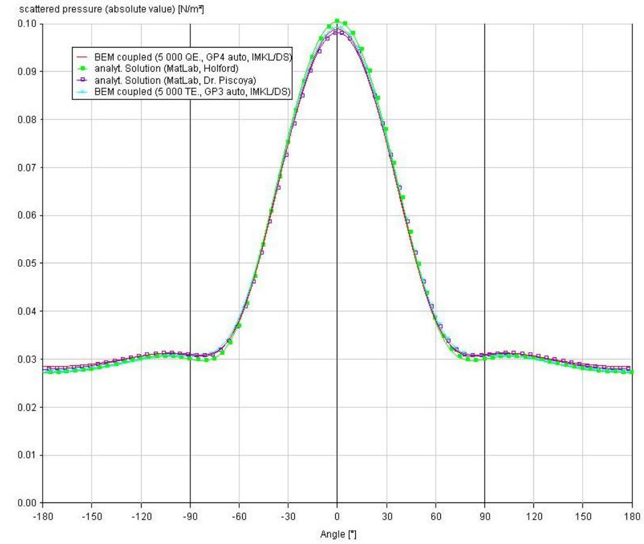

Figure 14: 1mm aluminium shell at 1 000 Hz, filled with air and surrounded by water

Here all solutions show a very good agreement.

Summary

Using the BEM-coupling method for multiple coupled cases, the accuracy of the results depends remarkably on the quality of discretization. The rough rule of thumb (6 elements per wavelength) appears no to be sufficient. To get almost the same accuracy as with triangular elements, we only need half of the number of quad elements. Using an automated Gauss adaption algorithm for near opposite elements ($\Delta l < l_{e,min}$) increases accuracy.

The critical case within our tests was the 1 cm shell where we get the largest differences in quality between analytical und numerical results.

Commercial applications based on elastic finite elements did not achieve comparable results within an acceptable time for shell widths $\Delta r < 2$ cm due to discretization limits.

Future focus

The multi-coupled test cases will be calculated using a finer discretization (> 10.000 elements) with an iterative solver to checkout the increase of accuracy. Also the modification of the coupling algorithm for “near” elements (i.e. using “shell elements”) has to be investigated.

References

[1] B. Nolte: „Randelementberechnungen und Nahfeld-messungen zur akustischen Fluid-Struktur-Interaktion“, PhD-Thesis, 1998, Universität der Bundeswehr Hamburg

[2] R. Burgschweiger, M. Ochmann, B. Nolte: „Berechnung der akustischen Rückstreustärke unter Berücksichtigung der Fluid-Struktur-Interaktion auf Basis einer BEM-BEM-Kopplung“, 34. Tagung der Dt. Akustischen Gesellschaft (DAGA2008), 2008, Dresden