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DIFFERENT METHODS FOR COMPUTING THE SOUND TRANSMISSION THROUGH FINITE, THIN BAFFLED PLATES

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In this paper, different approaches for calculating the sound transmission through thin finite plates under arbitrary (also dissipative) boundary conditions are investigated. First, the vibrations of the plate are determined by using a basis of polynomial functions. Second, a purely numerically based method, i.e. a coupled Finite-Element-Method (FEM) – Boundary-Element-Method (BEM) is applied. Finally, an iterative method which starts with the blocked- pressure approximation and takes into account the fluid load incrementally is developed. Results obtained with these methods are presented and discussed.

1. Introduction

In a joint DFG project, different approaches to calculate the sound transmission from plate-like structures are investigated. The entire numerical task consists of a fully coupled, fluid-structure-interaction problem. The computation of the exciting sound field, the plate motion and the radiated sound are subtasks that can be solved in different ways, for example, with analytical derivations, discretization methods (FEM or BEM), and modal or polynomial basis functions for the plate motion.

When these methods are applied to the coupled problem, a system of linear equations is obtained. This system is solved by a direct method like for example the Gaussian elimination method. In the middle and high frequency range, where the discretization must be fine enough to ensure at least six finite elements per wave length, a direct solution may be very time consuming. Therefore, suitable iterative techniques represent an interesting alternative. One possible iterative scheme improves the blocked-pressure approximation with successive corrections until convergence is obtained. In every step of the procedure, structure and fluids are treated separately and then the results are combined to go on to the next step. Unfortunately, it may happen that this approach will not converge for plates with weak damping. Therefore, methods for overcoming such instabilities and guaranteeing convergence need to be implemented.

2. Equation of motion of the plate

When sound waves hit a plate, it vibrates and in turn this vibration produces sound. The normal displacement of the plate *u* is responsible for the transmission of sound from one side to the other.

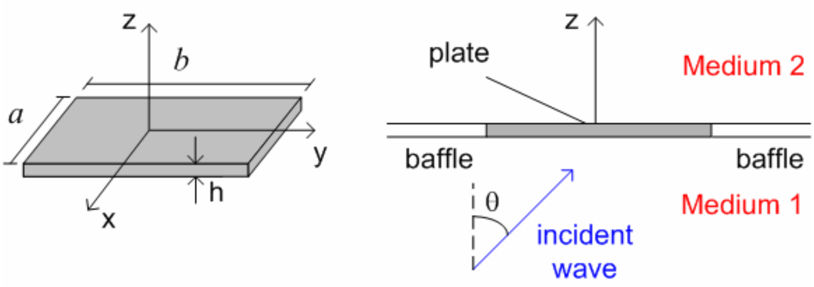

Figure 1. Sound transmission through a baffled plate.

To calculate *u*, a variational principle can be applied. Within this approach, the plate vibrations correspond to minimum values of the action functional *J* defined as

$$J = \int_{t_0}^{t_1} (T - V)dt + \int_{t_0}^{t_1} W dt\,, \tag{1}$$

where *T* and *V* are the kinetic and potential energies and *W* is the work of the external forces. For the plate, the model of Kirchhoff will be assumed. *T*, *V* and *W* are written as

$$T = \frac{1}{2}\int_{\Omega} \rho \left(\frac{\partial u}{\partial t}\right)^2 d\Omega \quad , \quad V = \frac{1}{2}\int_{\Omega} \sigma_{ij}(u)\varepsilon_{ij}(u)d\Omega \quad , \quad W = \int_{S} FudS\,, \tag{2}$$

where Ω and *S* are the volume and surface of the plate and σ_{ij} and ε_{ij} are the elements of the stress and strain tensors respectively.

If *u* is developed into a modal or polynomial basis ϕ_n and the infinite series is truncated after *N* terms

$$u = \sum_{n=1}^{N} a_n \phi_n\,, \tag{3}$$

the following equations for the coefficients a_n can be deduced [1]

$$\left(K_{qn} - \omega^2 M_{qn}\right) a_n = F_q \quad , \quad q, n=1, 2,..., N \tag{4}$$

K_{qn} is the stiffness matrix, M_{qn} the mass matrix, and F_q the vector of external forces.

2.1 Blocked-pressure approximation

The vector of external forces has two contributions, one part is coming from the sound sources, F_q^s, and other from the fluid load due to the sound radiation, F_q^r, $F_q = F_q^s + F_q^r$.

When the density of the fluid surrounding the plate is much smaller than the density of the plate, the term F_q^r is very small compared to F_q^s and can be neglected. Then, the vector of external forces is written as

$$F_q = P_q^{bp} = \int_S p^{bp} \phi_q dS \quad , \qquad p^{bp} = 2 p_{inc} \text{ (blocked pressure)} \tag{5}$$

where p_{inc} denotes the pressure of the incident wave. The coefficients a_n in the series (3) will be specified only by the exciting sound wave.

Under this assumption, the vibrations of the plate and the sound radiation are only coupled "one way" and displacement and sound radiation can be computed in two separate steps. The coefficients a_n are determined numerically solving Eq. (4), and u is computed using Eq. (3). The transmitted sound pressure is finally obtained by solving the Rayleigh integral

$$p = j\rho_2 \omega \int_S \dot{u} g_{H2} dS \tag{6}$$

with $\dot{u} = \frac{\partial u}{\partial t}$, $g_{H2} = \frac{e^{jk_2|\vec{x}-\vec{y}|}}{2\pi|\vec{x}-\vec{y}|}$ and $\left.\frac{\partial g_{H2}}{\partial n}\right|_{baffle} = 0$.

2.2 Inclusion of fluid load

The contribution of the fluid load to the vector of external forces can be expressed as

$$F_q^r = \int_S \Delta p^S \phi_q dS \quad \rightarrow \qquad F_q = P_q^{bp} + F_q^r \ , \tag{7}$$

where $\Delta p^S = p_1^S - p_2^S$ is the difference of the pressures at the the two surfaces of the plate due to the sound radiation. Inserting Eq. (7) in Eq. (4), the equation for the displacement vector is obtained

$$\left(K_{qn} - \omega^2 M_{qn} + j\omega\left(Z_{qn}^{(1)} + Z_{qn}^{(2)}\right)\right)a_n = P_q^{bp} \tag{8}$$

with $Z_{qn}^{(1)} = j\rho_1 \omega \int_S \int_S \phi_q g_{H1} \phi_n dS dS$ and $Z_{qn}^{(2)} = j\omega\rho_2 \int_S \int_S \phi_q g_{H2} \phi_n dS dS$.

The sound transmitted to the other side of the plate is given again by (6) .

3. Solution of the equation of motion

We present three methods to compute the displacement of the plate. The first two methods solve Eq. (8) using different basis functions. We refer to these methods as *direct approaches*, since the solution is obtained in one step. The third method finds the solution by applying an iterative scheme. Starting with certain approximate values, displacement and sound pressure are improved in successive steps until convergence is reached. We refer to it as the *iterative method*.

3.1 Semi-analytical solution (Woodcock 1995 [1])

If a polynomial basis of the form $\phi_{nm}(x, y) = \varphi_n(x)\psi_m(y)$, with functions $\varphi_n(x) = (2x/a)^n$ and $\psi_m(y) = (2y/b)^m$ is chosen [1], the displacement is written as

$$u = \sum_{nm} a_{nm} \phi_{nm} \quad , n,m=0,1,....$$

Note that now two indices are used to specify the order of the function. The elements of the matrices in Eq. (8) contain surface integrals of products of the polynomials and their first and second derivatives. The elements of mass and stiffness matrix can be calculated analytically in closed form, but the elements of the radiation impedances and forces can be expressed only as infinite series.

The elements of the mass matrix are given by

$$M_{pqnm} = \rho h \int_S \varphi_p(x)\psi_q(y)\varphi_n(x)\psi_m(y)dxdy \,.$$

The elements of the stiffness matrix can be defined as the sum of six contributions $K_{pqnm} = K^{(1)}_{pqnm} + K^{(2)}_{pqnm} + K^{(3)}_{pqnm} + K^{(4)}_{pqnm} + K^{(5)}_{pqnm} + K^{(6)}_{pqnm}$. Here, only the explicit form of $K^{(1)}_{pqnm}$ is shown, the other ones can be found in [1]

$$K^{(1)}_{pqnm} = \frac{Eh^3}{12(1-\nu^2)} \int_S \frac{\partial^2 \varphi_p(x)}{\partial x^2} \psi_q(y) \frac{\partial^2 \varphi_n(x)}{\partial^2 x^2} \psi_m(y)dxdy \,.$$

Arbitrary boundary conditions can be taken into account by adding an additional term to the stiffness matrix, $K^{(bc)}_{pqnm}$, which is defined as

$$K^{(bc)}_{pqnm} = \int_\Gamma K\varphi_p(x)\psi_q(y)\varphi_n(x)\psi_m(y)d\Gamma + \int_\Gamma C \frac{\partial\left(\varphi_p(x)\psi_q(y)\right)}{\partial n} \frac{\partial\left(\varphi_n(x)\psi_m(y)\right)}{\partial n} d\Gamma \,.$$

K and C are translational and rotational stiffness, and Γ is the contour of the plate. Dissipative boundary conditions can be also considered, if both parameters are defined as complex quantities in the form $K = K\left(1 - j\eta_K\right)$ and $C = C\left(1 - j\eta_C\right)$.

The radiation impedance $Z^{(1)}_{pqnm}$ can be developed into an infinite series

$$Z^{(1)}_{pqnm} = \rho_1 \omega k_1 \frac{a^2 b^2}{32\pi} \sum_i (-1)^i \frac{\left(k_1 a/2\right)^{i-1}}{(2i)!} \left[\frac{\left(k_1 a/2\right)}{2i+1} J^{(i)}_{pqnm} + jI^{(i)}_{pqnm} \right].$$

$J^{(i)}_{pqnm}$ and $I^{(i)}_{pqnm}$ are also series, each one with a number of terms depending on i. The expression for $Z^{(2)}_{pqnm}$ is similar, one has to change the index 1 for 2.

Finally, the elements of vector of P^{bp}_{pq} can be expressed as

$$P^{bp}_{pq} = 2\left(\frac{2}{a}\right)^p \left(\frac{2}{b}\right)^q I_p I_q \,,$$

where I_p and I_q can be computed using recurrence formulas. The formula for I_p is

$$I_p = \frac{1}{j\alpha}\left(\frac{a}{2}\right)^p \left(e^{j\alpha a/2} - (-1)^p e^{-j\alpha a/2}\right) - \frac{p}{j\alpha} I_{p-1} \quad , \quad \alpha = k_1 n_{ix} \,, \quad I_0 = \frac{2}{\alpha}\sin\left(\frac{\alpha a}{2}\right).$$

n_{ix} is the x-component of the direction vector of the incident wave. The expression for I_q is similar.

3.2 Numerical solution (FEM and BEM)

The displacement of the plate is computed by discretizing the plate and the basic equations. The meshes for FEM and BEM are not necessarily equal. In many cases, the structural wavelength is shorter than the acoustic wavelength and a finer discretization of the plate is needed. When this happens, a matching of both meshes has to be performed. This task can make the calculations more complicated. In this study, only one mesh for both methods has been used.

Combining a FE formulation for the motion of the plate and a BE formulation for the sound radiation, an equation for the displacement u, similar to Eq. (8), can be deduced [2]

$$\left(K - \omega^2 M + j\omega\left(Z_1 + Z_2\right)\right)u = Ap^{bp} \,, \tag{9}$$

where K is the stiffness matrix, M the mass matrix, Z_1 and Z_2 are the radiation impedances, A is the matrix of the surfaces of the elements, and p^{bp} the vector of the blocked pressure. If N is the number of elements, the size of the matrices is $N\times N$.

If the vector u is expanded in n eigenmodes ($n<N$), $u = \Phi d$, a smaller system of equations is solved. Usually, the modes in vacuum (dry modes) are considered and they are normalized with respect to the mass matrix. In the new system, the unknowns are the amplitudes of the eigenmodes (also called participation factors). The matrix equation to be solved is given by

$$\left(\omega_R^2 - \omega^2 I + j\omega\Phi^T\left(Z_1 + Z_2\right)\Phi\right)d = \Phi^T A p^{bp} . \tag{10}$$

ω_R^2 is the diagonal matrix $diag(\omega_1^2, \omega_2^2, ..., \omega_N^2)$ and Φ is the $N\times n$ matrix of the eigenvectors.

3.3 Iterative method

Starting from the blocked-pressure approximation, the displacement of the plate and the pressure difference at the surfaces of the plate are successively improved through combination from separate calculations of structure vibration and sound radiation until a sufficiently accurate value is achieved.

The movement of the plate due to the pressure excitation is given by

$$v = L_p\left(\Delta p_M + \Delta p_E\right) \ . \tag{11}$$

v is the normal velocity of the plate, L_p the operator of the motion of the plate in vacuum, Δp_M the pressure difference due to the motion of the plate, and Δp_E the pressure difference generated by the presence of sound sources (blocked pressure).

On the other hand, the pressure difference produced by the vibration of the plate is given by

$$\Delta p_M = -\left(L_F^{(1)} + L_F^{(2)}\right)v \ . \tag{12}$$

$L_F^{(1)}$ and $L_F^{(2)}$ are operators that describe the excitation of the plate by the sound waves in both fluid regions.

Based on the Eqs. (11) and (12), an iterative procedure can be constructed. The n-th double step of the procedure reads

$$\Delta p_M^{(n)} = -\left(L_F^{(1)} + L_F^{(2)}\right)v^{(n-1)}, \ v^{(n)} = L_p\left(\Delta p_M^{(n)} + \Delta p_E\right), \ n=0,1,... \ . \tag{13}$$

The starting value in (13) $v^{(-1)} = 0$ leads to the blocked pressure $\Delta p_M^{(0)} = 0$ and to the blocked-pressure approximation $v^{(0)} = L_p\left(\Delta p_E\right)$.

4. Results

A simply supported steel plate is studied. The dimensions of the plate in meters are 0.455×0.376×0.001 and at both sides air is assumed. The plate is excited by a plane wave which has a fixed incident angle of 45° and amplitude 1 Pa. The highest frequency considered is 800 Hz.

The eigenvalues and eigenmodes of the plate can be analytically calculated

$$\phi_{nm}(x,y) = \sin(n\pi/a)\sin(m\pi/b) ,$$

$$\omega_{nm}^2 = \frac{\pi^4 E h^2}{12\left(1-\nu^2\right)\rho}\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^2 .$$

The insulating properties of the baffled plate are quantified by the transmission loss R

$$R = 10\log\left(\frac{1}{\tau}\right), \quad \tau = \frac{\text{transmitted power}\,(W_t)}{\text{incident power}\,(W_i)}\,. \tag{14}$$

The incident sound power is given by the expression

$$W_i = \frac{|p_0|^2 ab\cos\theta}{2\rho_1 c_1} \tag{15}$$

and the transmitted sound power is computed using the formulas

$$W_t = \frac{\omega^2}{2}\mathrm{Re}\left(\sum_{pqnm} a_{pq} Z^{(2)}_{pqnm} a^*_{nm}\right) \qquad \text{(polynomial basis)}, \tag{16}$$

$$W_t = \frac{1}{2}\sum_{i=1}^{N}\mathrm{Re}\left(p_{2i}\dot{u}^*_i\right)S_i \qquad \text{(modal basis)}\,. \tag{17}$$

4.1 Comparison of the direct approaches

The eigenfrequencies and the curves of transmission loss were calculated and shown in Fig. 2.

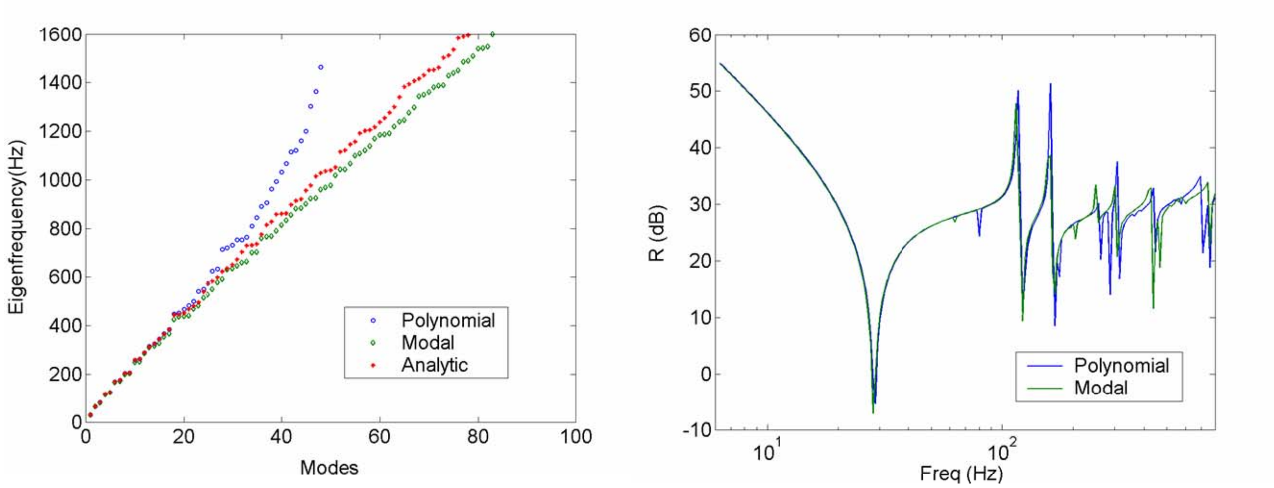

Figure 2. Results of both direct approaches

The eigenfrequencies obtained by the two direct methods agree very well with the analytical values up to about 400 Hz. The differences between the analytical and the numerical methods are relatively small. The deviations between the analytical and the polynomial-basis-method grow with increasing frequencies because a high accuracy is needed by computing the elements of the mass and stiffness matrices. The curves of R are practically the same up to 200 Hz. Beyond that frequency, small differences appear.

4.2 Convergence of the iterative method

The iterative method was implemented and applied to the steel plate. After only a few iterations, the values of the transmission loss near the eigenfrequencies diverged while the values for other frequencies converged. Fig. 3 illustrates the curves of R for the first 3 iterations. Only a narrow frequency range is captured to see the regions near two eigenfrequencies.

The lack of convergence is originated from the fact that the operator L_p becomes singular near the eigenfrequencies, if small damping or no damping at all is considered.

To overcome this problem, a technique based on the splitting method with an auxiliary matrix was investigated. Combining (11) and (12) the following equation is written

$$Z_s v = \Delta p_E - Z_F v \quad , \tag{18}$$

with $Z_s = L_p^{-1}$ and $Z_F = L_F^{(1)} + L_F^{(2)}$. It should be noted that L_p can be singular but not Z_s. Adding an auxiliary matrix Z_A to both sides of Eq. (18), a new iteration procedure can be built

$$v^{(n)} = \left(Z_s + Z_A\right)^{-1}\left(\Delta p_E - \left(Z_F - Z_A\right)v^{(n-1)}\right) \quad . \tag{19}$$

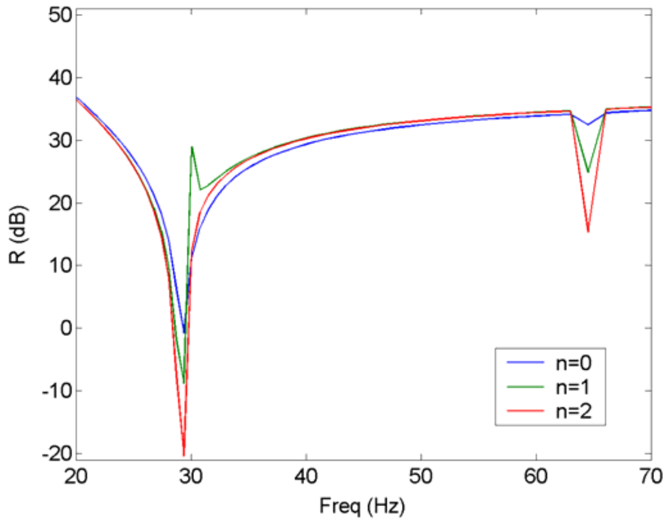

Figure 3. Transmission loss obtained with the iterative method

Now, a proper auxiliary matrix that assures convergence has to be found. Since the inverse of $\left(Z_s + Z_A\right)$ has to be performed, it is convenient to choose Z_A that makes $\left(Z_s + Z_A\right)$ diagonal in order to simplify the computation of the inverse. The only task remaining is to find the diagonal entries.

For the diagonal elements, the radiation impedance of the infinite panel multiplied by the surface area of the panel was used. With this selection, it was possible to reduce the spectral radius of $\left(Z_s + Z_A\right)$ to just below unity, so that the convergence was ensured. However, the rate of convergence was still too low. An alternative technique of improving iteration schemes is tested in [3].

5. Summary

In this paper, three methods to compute the transmission loss of thin baffled plates are presented. The first two are direct methods, where the first is semi-analytical and the second purely numerical. They provide similar results for the transmission loss in the frequency range studied. The third method involves an iterative procedure, but it does not converge near the eigenfrequencies of the plate. A modification of the scheme with the introduction of an auxiliary matrix leads to an improved convergence behaviour, but the procedure still needs some refinement.

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