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CALCULATION OF THE TRANSMISSION LOSS OF FINITE PLATES USING NUMERICAL MODAL ANALYSIS AND BEM

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In the present work, the transmission of sound through baffled thin plates is investigated. The curves of transmission loss in the frequency domain are computed with a coupled FEM-BEM approach. The method can be applied to plates with general properties under arbitrary boundary conditions. The variation of the transmission loss due to changes in surrounding media, boundary conditions and shape of the plate are presented. Simulated curves of the transmission loss of an orthotropic plate are compared with those of an isotropic plate.

1. Introduction

The estimation of sound insulation of plate-like structures is very important in room and building acoustics. Numerical predictions of the transmission loss allow a fast evaluation of modifications in the design of partition elements and enable a reduction of the number of required measurements. These estimations can be obtained through a combination of a numerical modal analysis for the plate and a boundary element (BE) formulation for the radiated sound. With a suitable discretization, the eigenmodes of the plate can be calculated using a finite element (FE) formulation and the sound radiation can be determined by computing a surface integral. For baffled flat plates, the BEM simplifies to a Rayleigh-Integral. The basic equations in both formulations are put together to obtain an equation for the plate displacement which is the common variable. If the displacement is expanded in a modal basis, the size of the system of equations to be solved can be reduced considerably. The combined method presented here is not limited to baffled plates. It can be extended to plates placed between sender and receiver rooms. In that case, the Helmholtz integral equation is used and the rooms have to be discretized as well, which implies an important increase in the size of the system matrices. To handle such big models, fast numerical algorithms, like for example the fast multipole method could be utilized. In the following sections, the method is described and the results of its application to different plate configurations are shown.

2. BE Formulation

We study the sound transmission of a sound wave with oblique incidence through a baffled finite thin plate. (Fig. 1). The media behind and in front of the plate can be assumed to be different.

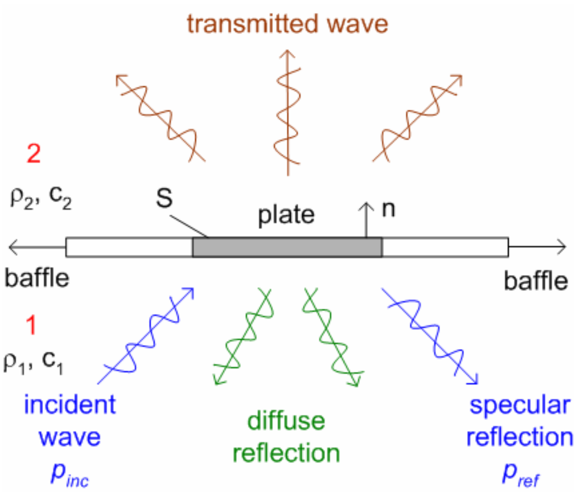


Figure 1. Transmission of a sound wave through a plate.

The plane wave in region 1 excites the plate and set it in motion. The vibration of the plate produces sound waves that propagate into both half-spaces. The sound pressures at both sides of the plate are given by the expressions [1]:

$$p_1 = p_{inc} + p_{ref} + \int_S \frac{\partial p_1^S}{\partial n} g_{H,1} dS \tag{1}$$

$$p_2 = -\int_S \frac{\partial p_2^S}{\partial n} g_{H,2} dS \tag{2}$$

where p_{ref} corresponds to the specular reflection. g_{H1} and g_{H2} are the Green's function in each half-space with a vanishing normal derivative at the baffle:

$$g_{H,1} = \frac{e^{-jk_1|\vec{x}-\vec{y}|}}{2\pi|\vec{x}-\vec{y}|} \quad , \quad g_{H,2} = \frac{e^{-jk_2|\vec{x}-\vec{y}|}}{2\pi|\vec{x}-\vec{y}|} \tag{3}$$

$$\frac{\partial g_{H,1}}{\partial n} = \frac{\partial g_{H,2}}{\partial n} = 0 . \tag{4}$$

We discretize the plate in N elements and suppose that all acoustic quantities are constant over each whole element. Assuming that the normal displacement of the plate is the same at both sides, we define the derivative of the pressures at the plate as:

$$\frac{\partial p_1^S}{\partial n} = \rho_1 \omega^2 u \quad , \quad \frac{\partial p_2^S}{\partial n} = \rho_2 \omega^2 u \ . \tag{5}$$

Inserting (5) in (1) and (2) we can write the integral equations in matrix form.

$$p_1^S = p_{inc}^S + p_{ref}^S + \rho_1 \omega^2 G_{H1} u \tag{6}$$

$$p_2^S = -\rho_2 \omega^2 G_{H2} u \tag{7}$$

p_1^S , p_2^S , p_{inc}^S , p_{ref}^S and u are vectors while G_{H1} and G_{H2} are the usual BEM-Matrices.

3. FE Formulation

The displacement of the plate u is the solution of the equation of motion

$$(K - \omega^2 M)u = F \tag{8}$$

K is the stiffness matrix, M the mass matrix and F the vector of the external force.

If F is set to 0, Eq. (8) merges into an eigenvalue problem and its solution are the eigenfrequencies ω_i und eigenmodes ϕ_i of the plate.

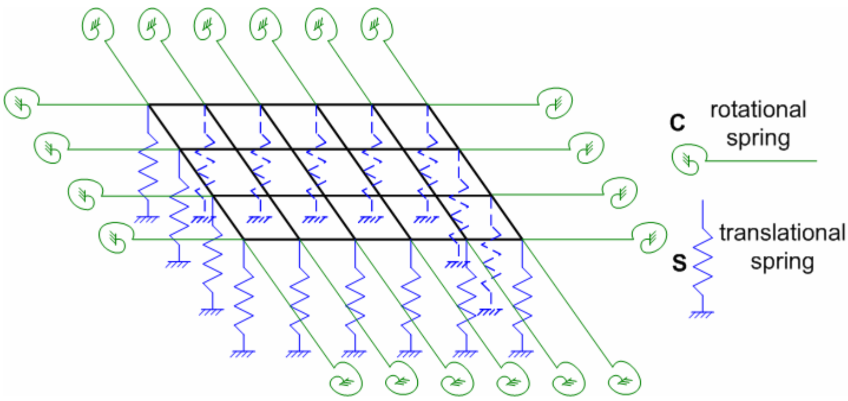


Figure 2. Elastic mounting of the plate.

The eigenmodes satisfy the boundary conditions. The usual boundary conditions are: free, simply supported und fixed borders. Other boundary conditions can be obtained by connecting the borders of the plate to the infinite wall with translational and torsional springs characterized by stiffness S and C respectively.

The displacement can be expanded in a series of n eigenmodes

$$u(x,y) = \sum_{i=1}^{n} d_i \phi_i(x,y) \quad \rightarrow \quad u = \Phi d \tag{9}$$

(Φ is the matrix of the eigenvectors). Therefore, the solution of (8) provides the amplitudes of the eigenmodes d_i. When the eigenvectors are normalized respective to the mass matrix, the following relations apply:

$$\phi_i^T M \phi_j = \delta_{ij} \ , \quad \phi_i^T K \phi_j = \omega_i^2 \delta_{ij} \ . \tag{10}$$

4. Coupled problem

Concerning the transmission of sound, the sound radiation and the vibration of the plate are coupled since the pressure oscillations excite the plate [2]

$$F_i = \left(p_{i1}^S - p_{i2}^S\right)A_i \qquad \text{for the } i\text{-th element} \tag{11}$$

where A_i corresponds to the surface of the i-th element.

Combining equations (6) - (8) and (11), we obtain the matrix equation for the displacement of the plate

$$\left(K - \omega^2 M + j\omega(Z_1 + Z_2)\right)u = A\left(p_{inc}^S + p_{ref}^S\right) \tag{12}$$

with $Z_1 = j\rho_1 \omega A G_{H1}$, $Z_2 = j\rho_2 \omega A G_{H2}$.

Inserting (9) in (12) and multiplying (12) from the left with Φ^T , one obtains the matrix equation for the unknown coefficients d

$$\left(\omega_R^2 - \omega^2 I + j\omega\left(\tilde{Z}_1 + \tilde{Z}_2\right)\right)d = \tilde{p}^{bp} \tag{13}$$

where ω_R^2 is a diagonal matrix $diag(\omega_1^2, \omega_2^2, ..., \omega_N^2)$ and

$$\tilde{Z}_{1,2} = \Phi^T Z_{1,2}\Phi \quad , \quad \tilde{p}^{bp} = \Phi^T A\left(p_{inc}^S + p_{ref}^S\right).$$

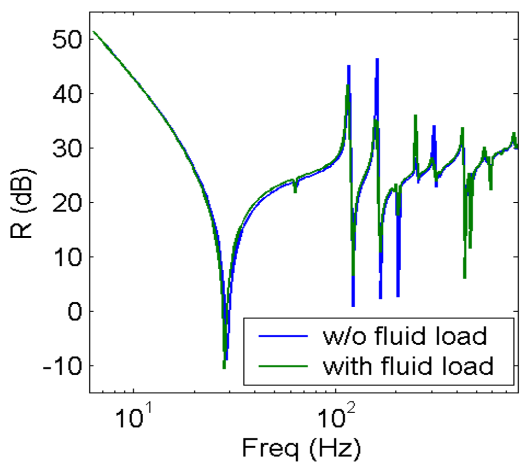
The matrices in (12) have dimensions $N\times N$ while the matrices in (13) dimensions $n\times n$ ($n<N$). Thus, the use of a modal basis can reduce significantly the dimension of the system to be solved, since the number of eigenmodes at low and middle frequencies is much smaller than the number of elements.

5. Results

The sound transmission is characterized by the transmission coefficient τ. The transmission coefficient is defined as the ratio of the incident to the transmitted sound power. In practice, the transmission loss or sound reduction index $R = -10\log\tau$ is used. We present the dependence of the curves of R versus frequency for different configurations.

5.1 Light vs. heavy medium

When the medium surrounding the plate is very light in comparison to the plate, the effect of the fluid load is small and can be neglected without making a big error. This is often the case when the medium is air. When the medium is heavy compared to the plate, the fluid load can not be neglected anymore. In this case, the fluid load shifts the minima of R to the low frequencies. This effect can be seen in Fig. 3.



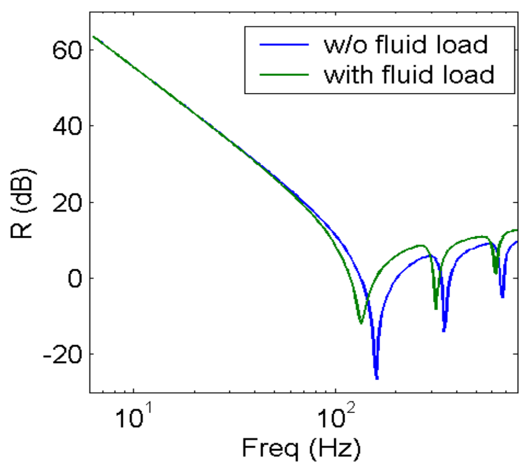


Figure 3. Transmission loss in air by a) heavy plate (left) and b) light plate (right).

5.2 Effect of boundary conditions (BCs)

The boundary conditions affect the transmission loss, because they determine the values of the eigenfrequencies. Varying the values of translational and rotational stiffness S and C, arbitrary BCs can be imposed. The usual BCs can be obtained by choosing S and C as shown in Table 1:

Table 1. Elastic mounting.

Boundary condition	*S*	*C*
Free	0	0
Simply supported	>>1	0
Fixed	>>1	>>1

In Fig. 4, the deformations of the plate for two different modes and three values of *S* are presented. In both cases *C*=0. It can be seen that for small *S*, the border of the plate is also deformed and for the biggest *S* the border barely moves, being near to the simply supported BC.

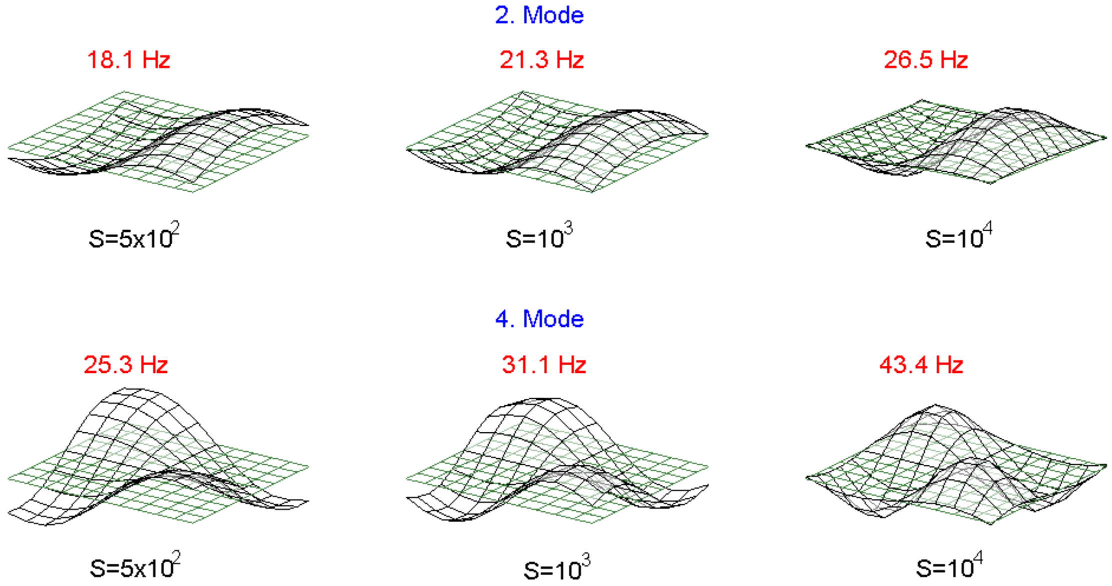


Figure 4. Eigenmodes of the plate for different values of *S*.

The curves of *R* for different values of *S* and *C* are illustrated in Fig. 5. In the first picture, *C*=0, and in the second one, *S* has high values.

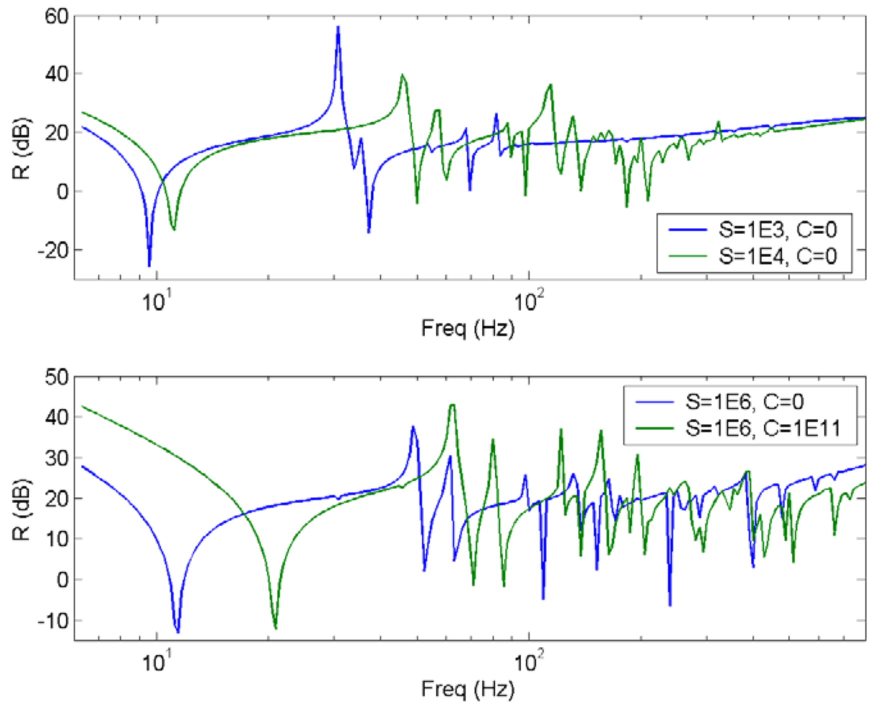


Figure 5. Transmission loss of a steel plate under different BCs.

5.3 Effect of the shape of the plate

To examine the effect of the shape of the plate, the transmission loss curves of a rectangular and an ellipsoidal plate having the same area are compared. The dimensions in meter of the rectangular plate are 0.455×0.376×0.001. For the ellipsoidal plate, the ratio of the large to the small axis is the same as the ratio of large to small side of the rectangular plate.

The curves of *R* are shown in Fig. 6 for simply supported and fixed BCs.. The rectangular plate has a superior insulation at low frequencies below a certain frequency f_e , which is lower than its first eigenfrequency. Beyond f_e, the ellipsoidal plate possesses a higher transmission loss. The same behaviour is seen with respect to both BCs.

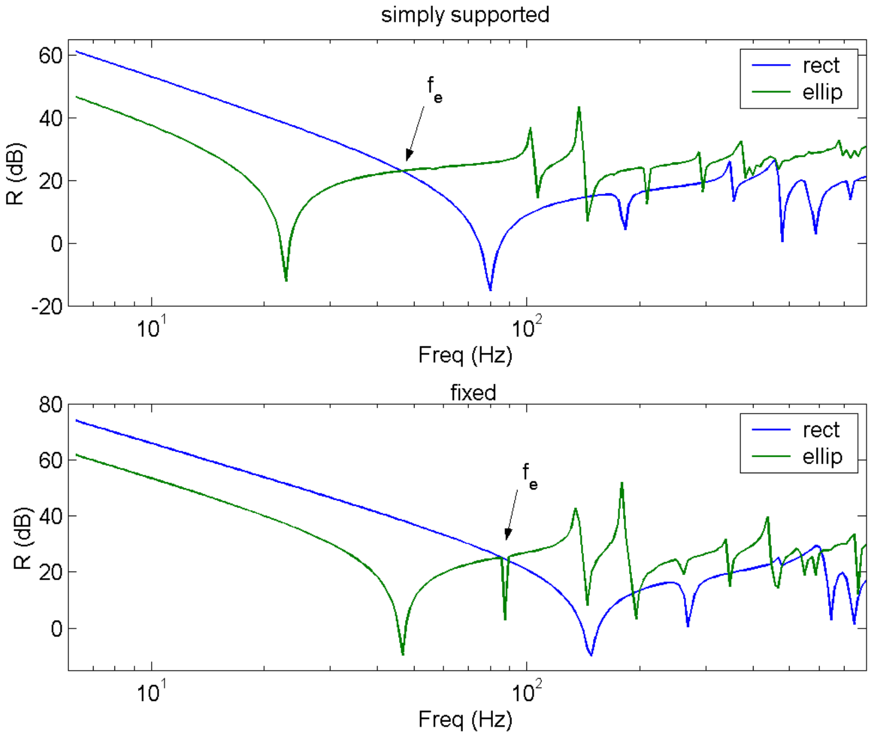


Figure 6. Comparison of *R* of a rectangular and ellipsoidal steel plate.

5.4 Isotropic vs. orthotropic plate

The material properties of orthotropic plates are different in two orthogonal directions. These materials are characterized by five parameters: E_x, E_y, v_x, v_y and G_{xy} , where E_x and E_y are the Young's modulus, v_x and v_y the Poisson's ratios and G_{xy} the shear modulus. However, only four of them are independent, because the following relation holds $E_x v_x = E_y v_y$. In contrast, isotropic plates are defined by three parameters, *E*, *v* and *G* where only two are independent, since $G=E/2(1+v)$. The eigenfrequencies of simply supported isotropic and orthotropic plates can be calculated analytically by the formulas [3]:

$$f_{k,l} = \frac{\pi}{2}\sqrt{\frac{B}{m}}\left(\frac{k^2}{a^2}+\frac{l^2}{b^2}\right) \qquad \text{(isotropic case)}$$

$$f_{k,l} = \frac{\pi}{2\sqrt{m}}\left(\frac{k^4}{a^4}B_x+\frac{l^4}{b^4}B_y+\frac{2k^2l^2}{a^2b^2}B_{xy}\right)^{1/2} \qquad \text{(orthotropic case)}$$

A comparison of the transmission loss of an isotropic plate and an orthotropic plate was performed. The parameters used for the calculations are listed in Table 2:

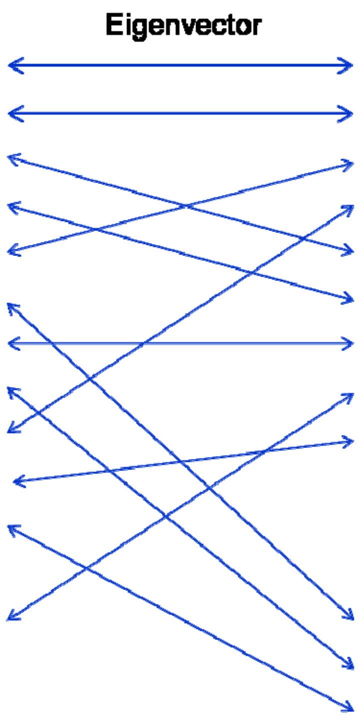
Table 2. Material parameters.

Material	E_x (Pa)	E_y (Pa)	v	G_{xy} (Pa)	ρ (Kg/m^3)
isotropic	7.6×10^{10}	-	0.34	-	1360
orthotropic	7.6×10^{10}	0.6×10^{10}	0.34	0.2×10^{10}	1360

In Table 3, the first 15 eigenfrequencies of both plates are listed. The modes with the same form of the deformation are connected with an arrow.

Table 3. Eigenfrequencies of isotropic and orthotropic plates.

Isotropic		Orthotropic	
Mode	Freq (Hz)	Mode	Freq (Hz)
1	85.1	1	51.3
2	187.9	2	67.6
3	236.5	3	105.5
4	334.8	4	165.3
5	359.6	5	194.3
6	489.1	6	202.4
7	499.6	7	222.6
8	581.0	8	245.4
9	600.3	9	261.1
10	731.9	10	322.3
11	735.2	11	344.6
12	842.9	12	407.2
13	910.1	13	432.3
14	927.7	14	436.3
15	953.8	15	446.1



The comparison of the curves of the transmission loss for simply supported and fixed plates are shown in Fig. 7.

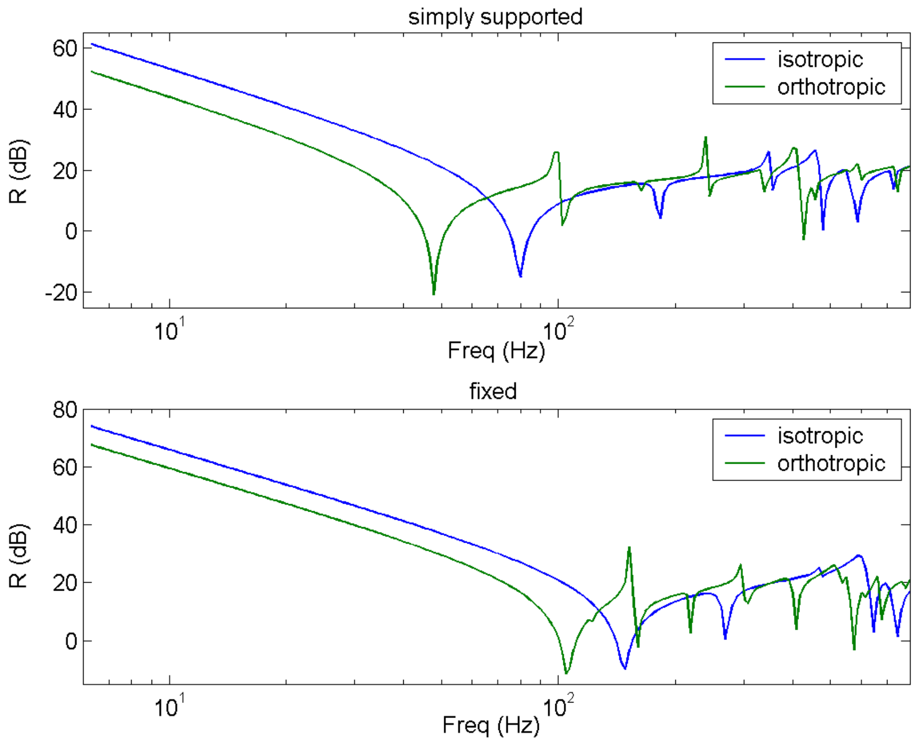


Figure 7. Comparison of *R* of a isotropic and orthotropic plate.

The isotropic plate has a better insulation than the orthotropic plate at low frequencies up to about the first eigenfrequency and beyond it both curves are similar. Only in a small region around the first isotropic eigenfrequency, the orthotropic plate has a higher transmission loss.

6. Summary

A numerical method based on the coupling of FEM and BEM calculations was developed to predict the sound transmission loss of thin baffled plates. The method can be applied to plates with general properties e.g. plates made of isotropic and orthotropic materials. The displacement of the plate is developed into the eigenmodes of the plate in vacuum which are previously computed by using the FEM. The participation coefficients, i.e. the amplitudes of the eigenmodes are found by solving the equation of motion of the plate including the fluid load determined by the BEM. The effect of the boundary conditions on the transmission loss was examined. The method is able to treat arbitrary BCs, which can be simulated by adding elastic elements to the border of the plate.

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