A Multi-Level Fast Multipole BEM-Method for computing the sound field in rooms

Updated paper due to error corrections within the results for rooms having impedance boundary conditions

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Summary

The Multi-Level Fast Multipole Method (MLFMM) allows the computation of acoustical problems where the discretized models of the corresponding structures may consist of a huge number of elements.

The required calculation time and the memory requirements are much less when compared with conventional boundary element methods because the algorithm uses a level-based composition of the potentials from different point sources to acoustic multipoles, which highly accelerates the computation of the matrix-vector-products required.

The MLFMM will be applied to room acoustical problems. Results for simple-shaped rectangular rooms equipped with different kinds of boundary conditions like for example, different impedance boundary conditions at each wall of the room, will be compared with respect to accuracy and solving time with analytical solutions and results based on conventional BEM-based calculations.

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1. Fast Multipole Method

The Multi-Level Fast Multipole Method (MLFMM) describes a fast algorithm to accelerate the matrix-vector-product which is required for the iterative solution of for BEM-based calculations without ever assembling the complete matrix.

The method is suited for big problems where the interactions between huge numbers of source (N_x) and destination points (N_y) must be considered.

The decrease of required interactions can be identified by comparing figures 1 and 2.

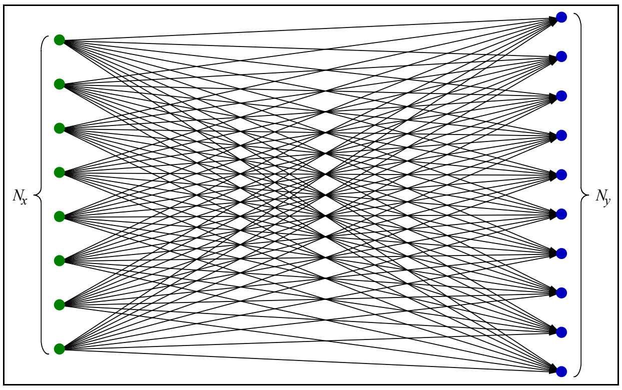

Figure 1. interaction scheme for conventional calculations

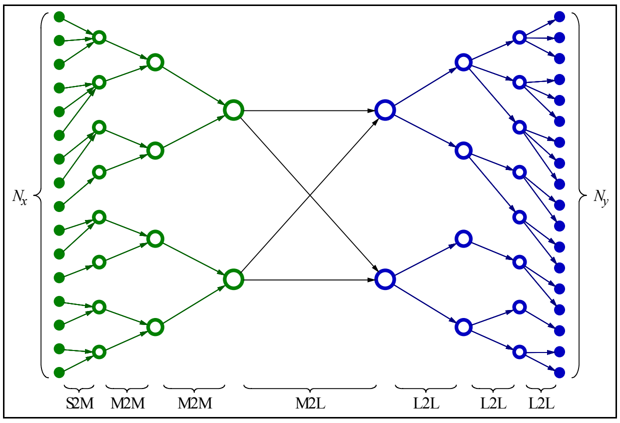

Figure 2. interaction scheme for a multi-level cluster-based calculation (using a maximum cluster level of $N_{lc,max} = 3$)

Within the three-dimensional case the clusters are represented by cubic boxes of different sizes, as shown in figure 3.

Each box contains a set of four so-called interaction lists (L_1 … L_4) which are describing the near-field and neighbourhood relations of a box and which are used to calculate the cluster interactions. Details of this algorithm and its implementation may be found in [3] and [4].

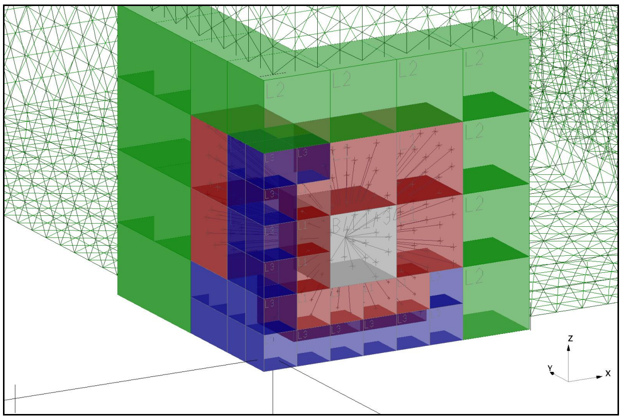

Figure 3. example of the MLFMM boxing algorithm at one corner of the room

A MLFMM-code for calculating the sound scattered from objects within fluids which supports the use of different kinds of solving methods and solvers was developed during a former project [1, 2]. This code was extended to interior problems combined with impedance boundary conditions.

2. Results for a room w/o interior wall

An air-filled room of 4 × 3 × 2 m was used in this first simple test case. A monopole source (◉) is placed in the room at [0.925 1.5 1] m. For comparability reasons, the model was build using COMSOL (407,200 finite elements) and the surface mesh (19,300 boundary elements) was exported for the BEM-based calculations.

The maximum border length of l_{max} = 0.1 m fits the 1/6 λ-condition up to a frequency of 500 Hz (λ_{air} = 0.686 m).

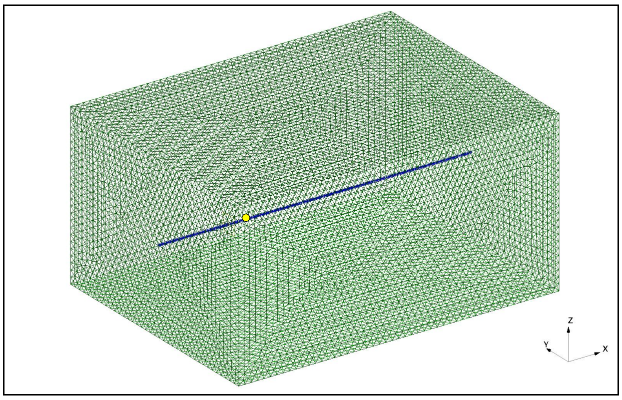

Figure 4. Room 4 x 3 x 2 m, containing a line of 391 field points in x-direction (see figure 7)

a) 200 Hz, rigid walls

Used abbreviations:

Δt_s	solving time
IMKL	Intel Math Kernel Library (direct solver)
GMRES	iterative Solver
MUMPS	sparse iterative Solver
N_{iter}	number of iterations
N_{elem}	number of elements (finite or boundary)
e	resulting iteration error (rel. residuum)

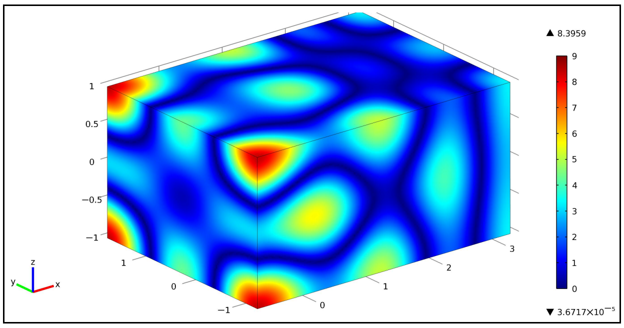

Figure 5. FEM (COMSOL), N_{elem} = 407,200, 200 Hz
Δt_s: 312 s (MUMPS)

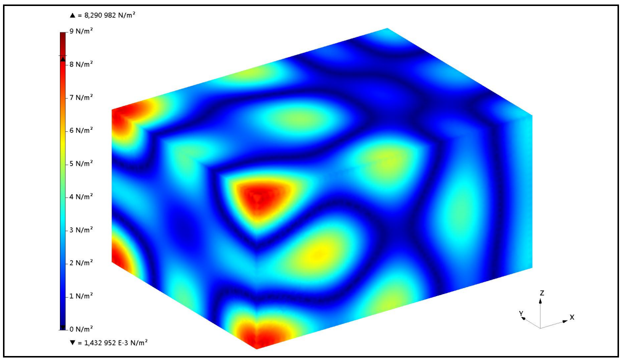

Figure 6. BEM, N_{elem} = 19,300, 200 Hz
Δt_s: 192.1 s (IMKL)
44.3 s (GMRES, N_{iter} = 65, $e < 10^{-5}$)
27.7 s (MLFMM, N_{iter} = 65, $e < 10^{-5}$)

The surface pressure (absolute value) for the FEM- and BEM-based results (figures 5 and 6) agrees very well and there are no significant differences between the direct and iterative solvers and an analytical solution at the field points inside the room.

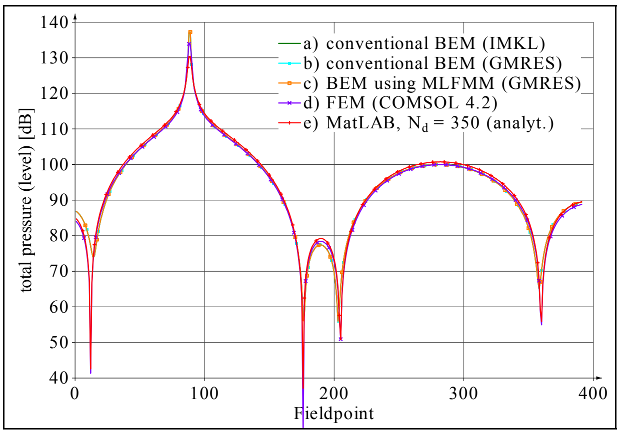

Figure 7. Sound pressure level at a line of 391 field points inside the room (no significant differences between all solutions)

b) 500 Hz, with impedance $Z=\rho c$ at the ceiling and the back wall

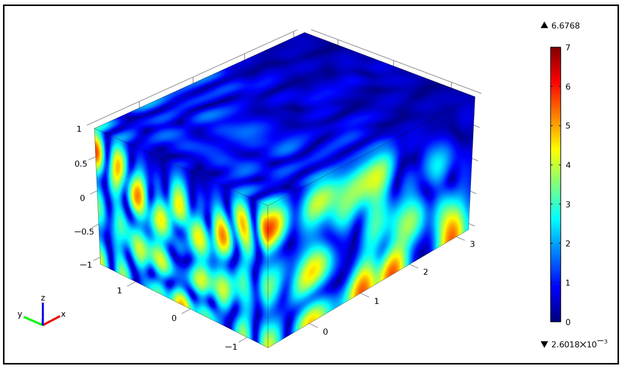

Figure 8. FEM (COMSOL), N_{elem} = 407,200, 500 Hz, with impedance
Δt_s: 229 s (MUMPS)

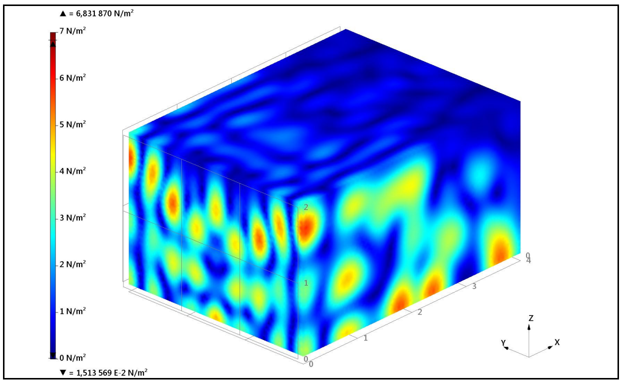

Figure 9. BEM, N_{elem} = 19,300, 500 Hz, with impedance
Δt_s: 198.8 s (IMKL)
48.1 s (GMRES, N_{iter} = 68, $e < 10^{-5}$)
51.6 s (MLFMM, O_{mp} =13, N_{iter} = 68, $e < 10^{-5}$)

Here the MLFMM-based solution has some visible differences at the “quieter” parts of the room when using a “lower” multipole order.

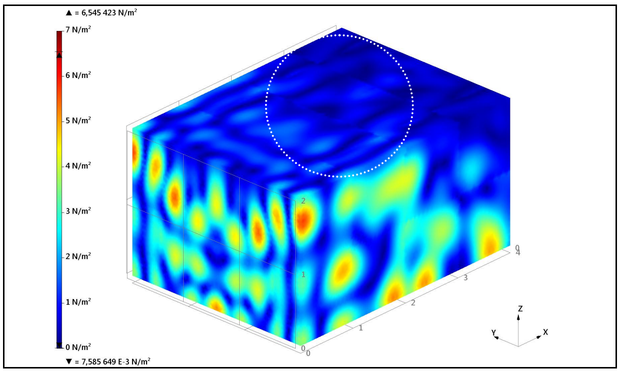

Figure 10. BEM using MLFMM, some small but visible differences compared with figure 9
Δt_s: 31.1 s (MLFMM, O_{mp} = 10, N_{iter} = 68, $e < 10^{-5}$)

3. Results for a room containing an interior dividing wall

The room was modified using a dividing wall with a thickness of 0.2 m and a depth of 2 m. The ceiling has an impedance of $Z = \rho c$, all other walls are rigid. The monopole source (☉) resides within the left part of the room. The FEM mesh consists of 3,197,500 elements and the resulting surface mesh needs 56,400 elements using a maximum border length of l_{max} = 0.05 m, suitable for 1 kHz.

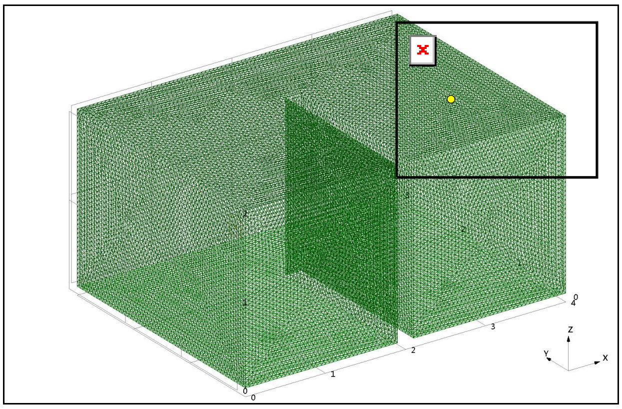

Figure 11. Room 4 × 3 × 2 m with dividing wall

a) 500 Hz, with impedance $Z=\rho c$ at the ceiling

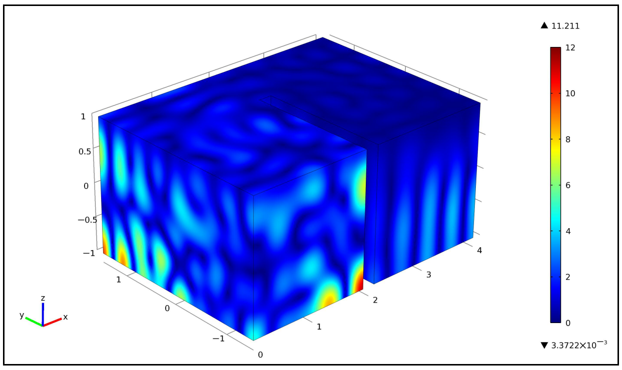

Figure 12. FEM (COMSOL), N_{elem} = 3,197,500
Δt_s:15,050 s (≈ 4:11 h, MUMPS)

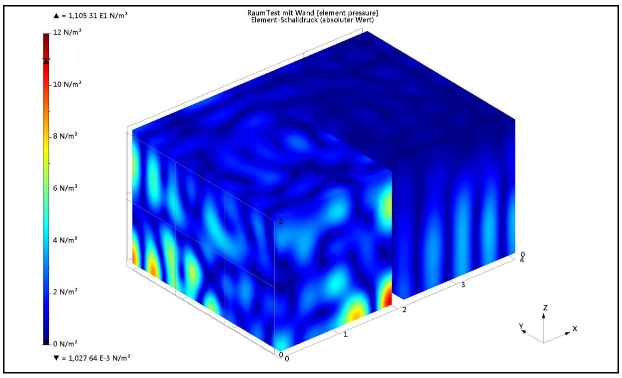

Figure 13. BEM, N_{elem} = 56,400, 500 Hz
Δt_s: 6,231 s (≈ 1:43 h, IMKL)
806 s (GMRES, N_{iter} = 170, $e < 10^{-5}$)

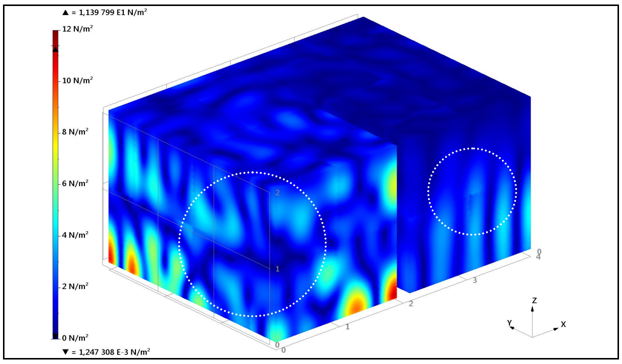

Figure 14. BEM with MLFMM, small differences
Δt_s: 361 s (GMRES, $N_{iter} = 163$, $e < 10^{-5}$)

b) 1 kHz, with impedance *Z=ρc* at the ceiling

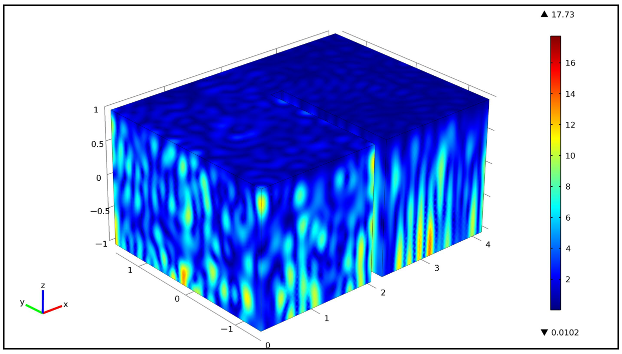

Figure 15. FEM (COMSOL), $N_{elem} = 3{,}197{,}500$, 1 kHz
Δt_s:15,248 s (≈ 4:14 h, MUMPS)

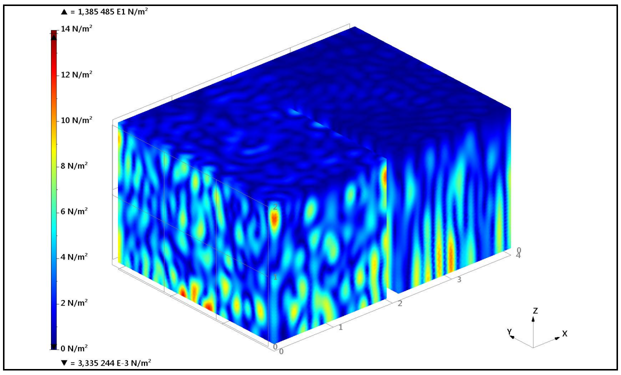

Figure 16. BEM, $N_{elem} = 56{,}400$, 1 kHz
Δt_s: 6,194 s (≈ 1:43 h, IMKL)
2,232s (≈ 0:37 h, GMRES, $N_{iter} = 652$, $e < 10^{-5}$)

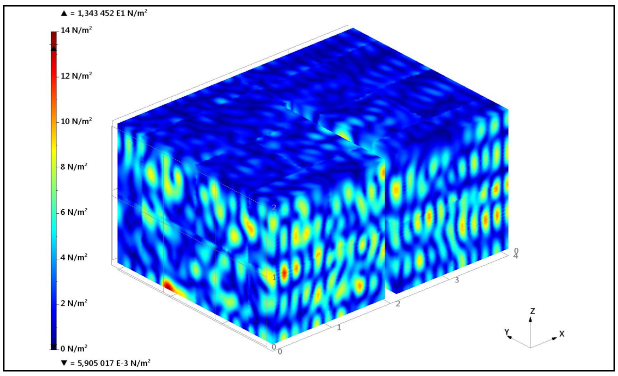

Figure 17. BEM with MLFMM, big differences, boxing structure limits are partially visible
Δt_s: 2,075 s (GMRES, $N_{iter} = 491$, $e < 10^{-5}$)

4. Conclusions

The results achieved have demonstrated that the conventional BEM compared with the FEM method gives comparable results at lower solving times. A significant performance advantage can be achieved when treating complex structures and different kinds of boundary conditions.

The Fast Multipole Method also seems to be applicable but has differences in quality at higher frequencies above 500 Hz especially in “quieter” regions of the structure due to the method-based errors when computing the matrix vector product.

Additional investigations seem to be necessary in optimizing the MLFMM-code and a level-based adaption of the multipole order.

An adequate preconditioning of the iterative solver is needed to reduce the number of iterations because a good convergence is a precedent condition for a successful application of the MLFMM.

Acknowledgements

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Literature

[1] R. Burgschweiger, I. Schäfer and M. Ochmann: „A Multi-Level Fast Multipole Algorithm (MLFMM) for calculating the Sound scattered from Objects within Fluids“, Proceedings of 20th International Congress on Acoustics, ICA 2010, Sydney, Australia

[2] M. Ochmann, R. Burgschweiger and C. Steuck: “Numerical experiments for testing the convergence of the acoustical Fast Multipole Method”, Proceedings of the 1st EAA Congress on Sound and Vibration (EuroRegio 2010), Ljubljana, Slovenien

[3] H. Cheng, L. Greengard and V. Rokhlin: “A Fast Adaptive Multipole Algorithm in Three Dimensions”, Journal of Computational Physics, 1999, Vol. 155, P. 468-498

[4] N. A. Gumerov and R. Duraiswami: “Fast Multipole Methods for the Helmholtz Equation in three dimensions”, 2004, Elsevier Books, ISBN 0-08-04431-0