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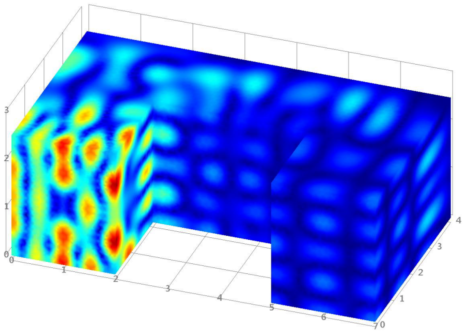
A Multi-Level Fast Multipole BEM-Method for computing the sound field in rooms



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Abstract & presentation contents

Abstract

The Multi-Level Fast Multipole Method (MLFMM) allows the computation of acoustical problems where the discretized models of the corresponding structures may consist of a huge number of elements.

The required calculation time and the memory requirements are much less when compared with conventional boundary element methods because the algorithm uses a level-based composition of the potentials from different point sources to acoustic multipoles, which highly accelerates the computation of the matrix-vector-products required.

The MLFMM will be applied to room acoustical problems. Results for simple-shaped rectangular rooms equipped with different kinds of boundary conditions like for example, different impedance boundary conditions, will be compared with respect to accuracy and solving time with results based on conventional BEM-based calculations and a commercial FEM-based application.

Presentation contents:

- Basics of the Multi-Level Fast Multipole Method
- 3 test cases for different room geometries and boundary conditions
- Conclusion

Basics of the Multi-Level Fast Multipole Method

The Multi-Level Fast Multipole Method describes a fast algorithm to accelerate the matrix-vector-product evaluation which is required for the iterative solution of BEM-based numerical calculations without ever assembling the complete matrix.

The method is suited for "big" physical problems where the interactions between huge numbers of source (N_x) and destination points (N_y) must be considered.

The decrease of required interactions can be identified by comparing figures 1 and 2.

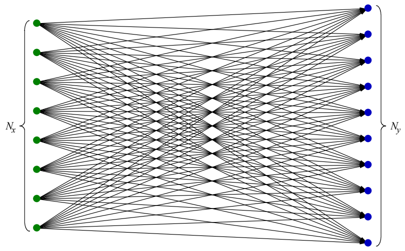


Figure 1: interaction scheme for conventional calculations

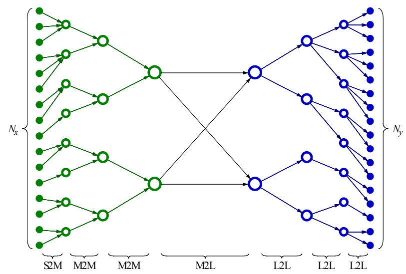


Figure 2: interaction scheme for a multi-level cluster-based calculation (using a maximum cluster level of $N_{lc,max} = 3$)

Within the three-dimensional case the clusters are represented by cubic boxes of different sizes which contains the source and destination points on the boundary (mesh).

Each box contains a set of four so-called interaction lists (L1 … L4, as shown in figure 3) which are describing the near-field and neighborhood relations of a box and are used to define the cluster interactions required.

Details of this algorithm and its implementation details may be found in [3] and [4].

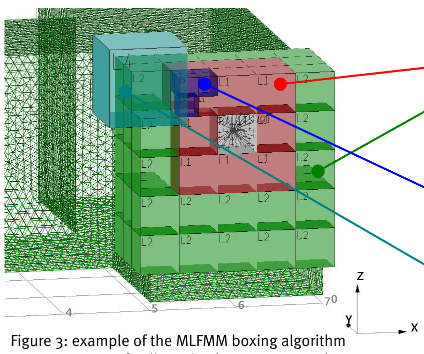
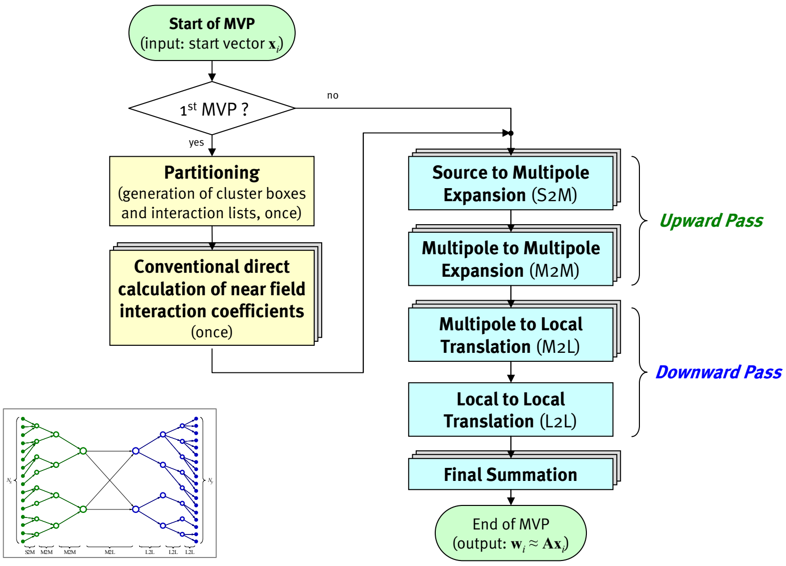


Figure 3: example of the MLFMM boxing algorithm at one corner of a discretized room BEM mesh

Interaction lists of a box B:

- **L1:** contains all immediate neighbors of **B** ("near field")
- **L2:** contains all children of **B**'s parents neighbors, which are not immediate neighbors of **B** (all boxes in L2 have the same size as **B**)
- **L3:** contains all children of **B**'s neighbors, which are not immediate neighbors of **B**, but whose parent is a neighbor of **B**
- **L4:** contains all boxes, which have **B** in their own L3 list (they are always childless and bigger than **B**)

MLFMM algorithm flow chart



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Implementation / Test environment

Test environment

A high performance C++-based object oriented code for calculating the sound scattered from objects within fluids that supports the use of different kinds of parallelized solving methods and solvers including a Multi-Level Fast Multipole algorithm was developed during a previous project [1, 2].

This code was extended to interior problems combined with impedance boundary conditions.

Results were calculated for simple room geometries using

- A conventional matrix based BEM-method with a direct linear solver (Intel Math Kernel Library, IMKL)
- A conventional matrix based BEM-method with an iterative solver (General Minimal Residuum, GMRES, without preconditioning)
- A MLFMM-based solution with an iterative solver (GMRES , without preconditioning)
- A commercial FEM-based application (COMSOL Multiphysics 4.x)

Used hardware for calculations: Intel Dual-XEON Workstations
a) matrix order ≤ 50,000: 12 cores, 2.66 GHz, 48 GB RAM, Microsoft Windows 7 Prof.
b) matrix order ≤ 75,000: 8 cores, 2.66 GHz, 96 GB RAM, Microsoft Windows 7 Prof.

Test case 1: Simple room

An air-filled room of 4 × 3 × 2 m was used in this first simple test case. A monopole source (○) is placed in the room at [0.925 1.5 1] m as shown in figure 4.

For comparability reasons, the model was build using COMSOL (407,200 finite elements) and the surface mesh (19,300 boundary elements) was exported for the BEM-based calculations.

The maximum border length of l_{max} = 0.1 m fits the 1/6 λ-condition up to a frequency of 500 Hz (λ_{air} = 0.686 m).

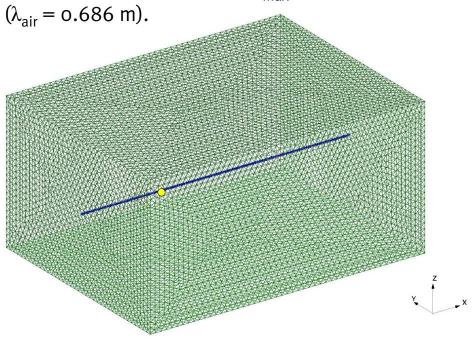

Figure 4: Room 4 × 3 × 2 m, containing a line of 391 field points in x-direction

Used abbreviations:

Δt_s	solving time
IMKL	Intel Math Kernel Library (direct linear solver)
GMRES	iterative Solver
MUMPS	sparse iterative Solver (used by COMSOL)
N_{iter}	number of iterations
N_{elem}	number of elements (finite or boundary)
e	resulting iteration error (relative residuum)

Simple room, rigid walls, f= 200 Hz

FEM

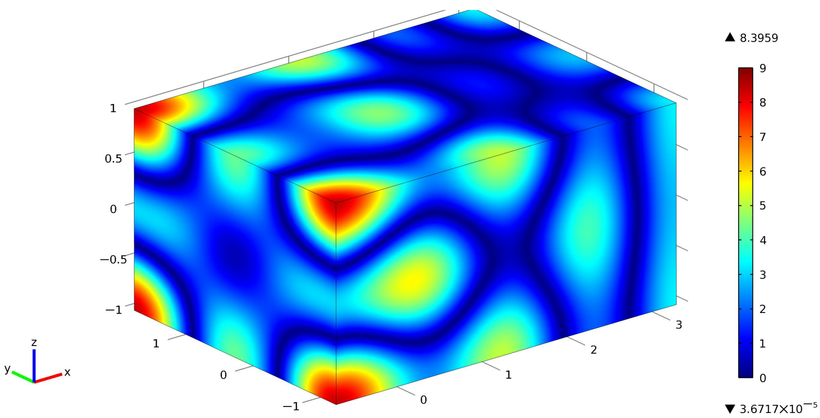

Figure 5: N_{elem} = 407,200 (FEM, COMSOL)
Δt_s: = 312 s (MUMPS)

abs(p_{surf})

Simple room, rigid walls, *f* = 200 Hz

BEM

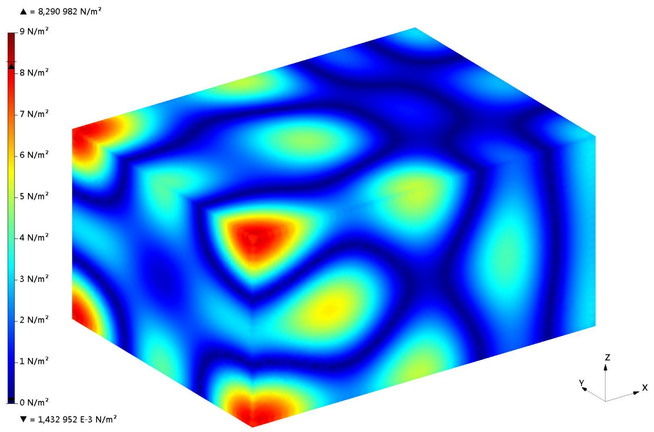

Figure 6: N_{elem} = 19,300 (BEM , $S_{mx} \approx 5.7$ GB)
Δt_s: = 192.1 s (direct, IMKL)
44.3 s (iterative, GMRES, $N_{iter} = 65$, $e < 10^{-5}$)
27.7 s (iterative, MLFMM (P20/M8/U9), $N_{iter} = 65$, $e < 10^{-5}$)

abs(p_{surf})

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Test cases and results

Simple room, rigid walls, *f* = 200 Hz

BEM

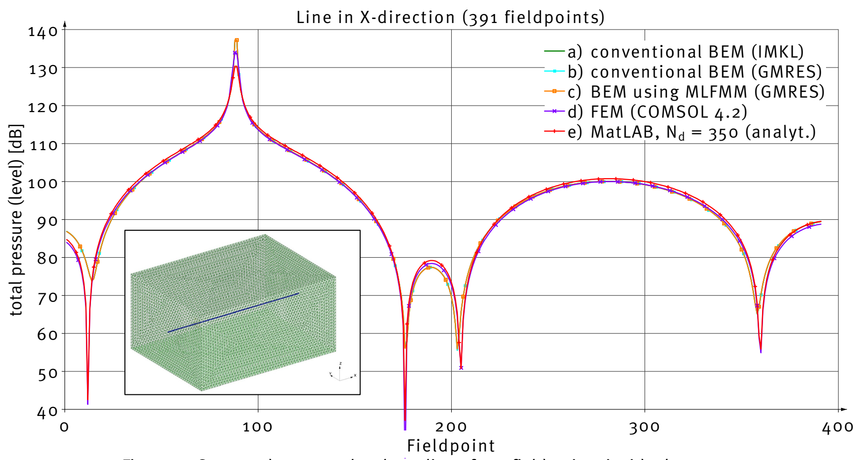

Figure 7: Scattered pressure level at a line of 391 field points inside the room
(no significant differences between all solutions)

Simple room, impedance $Z=\rho c$ at ceiling & back wall, $f=$ 500 Hz **FEM**

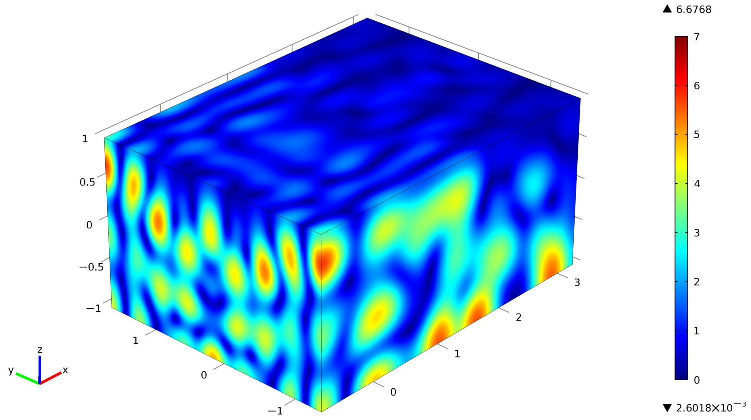

Figure 8: N_{elem} = 407,200 (FEM, COMSOL , $S_{mem} \approx$ 15.5 GB)
Δt_s: = 229 s (MUMPS)

abs(p_{surf})

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Test cases and results

Simple room, impedance $Z=\rho c$ at ceiling & back wall, $f=$ 500 Hz **BEM**

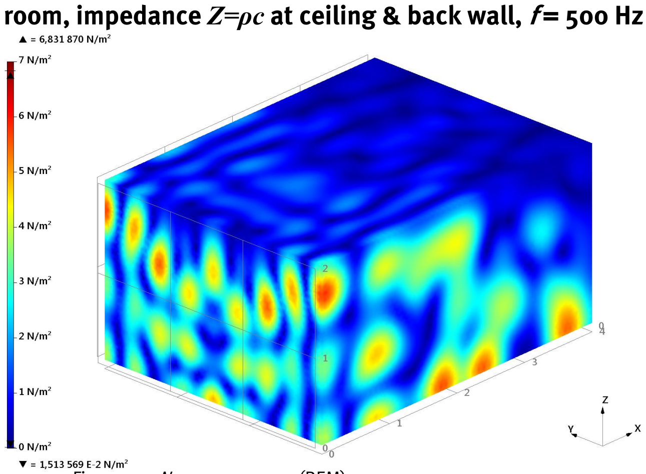

Figure 9: N_{elem} = 19,300 (BEM)
Δt_s: = 198.8 s direct (IMKL)
48.1 s iterative, GMRES, N_{iter} = 68, e < 10^{-5}
51.6 s iterative, MLFMM (P50/M13/U14), N_{iter} = 68, e < 10^{-5}

abs(p_{surf})

Simple room, impedance $Z=\rho c$ at ceiling & back wall, f= 500 Hz **MLFMM**

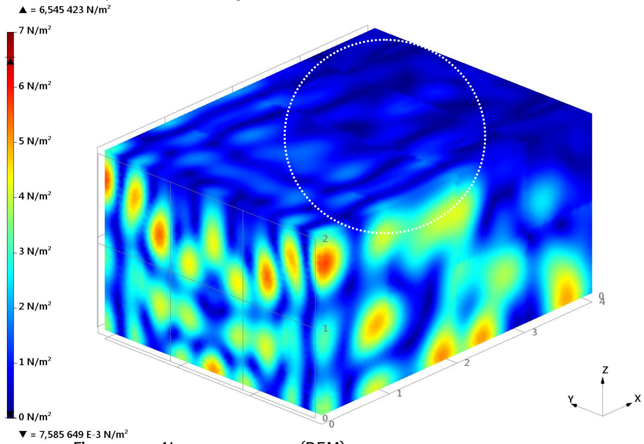

Figure 10: N_{elem} = 19,300 (BEM)
Δt_s: = 31.1 s iterative, MLFMM (P50/M10/U11), N_{iter} = 68, e < 10^{-5}

abs(p_{surf}) Some small but visible differences when using this multipole order (see marker)

Test case 2: Room containing an interior dividing wall

The room was modified using a dividing wall with a thickness of 0.2 m and a depth of 2 m. The ceiling has an impedance of $Z = \rho c$, all other walls are rigid.

The monopole source (●) resides within the left part of the room. The FEM mesh consists of 3,197,500 elements and the resulting surface mesh needs 56,400 elements using a maximum border length of l_{max} = 0.05 m, suitable for 1 kHz.

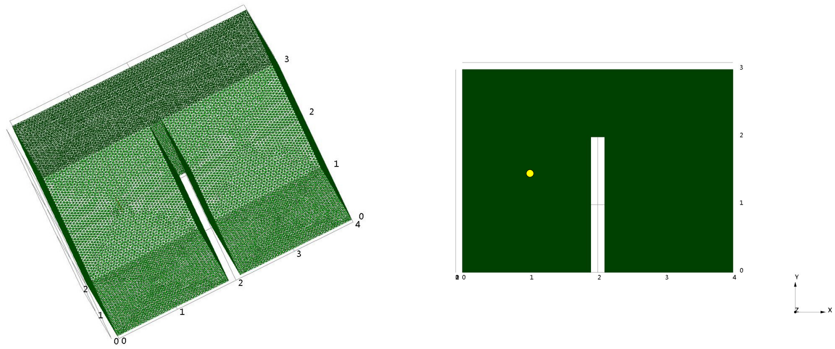

Figure 11: Room 4 × 3 × 2 m with dividing wall

Room with wall, impedance $Z = \rho c$ at ceiling, $f = 500$ Hz

FEM

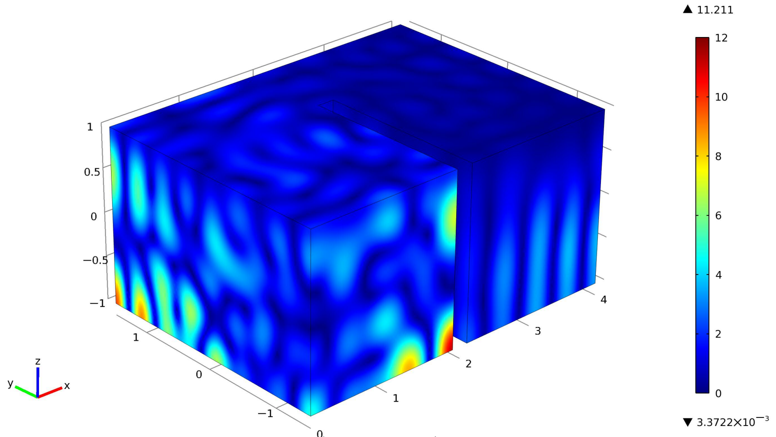

Figure 12: N_{elem} = 3,197,500 (FEM, COMSOL)
Δt_s: = 15,050 s (MUMPS), ≈ 4:11 h

abs(p_{surf})

Room with wall, impedance $Z = \rho c$ at ceiling, $f = 500$ Hz

BEM

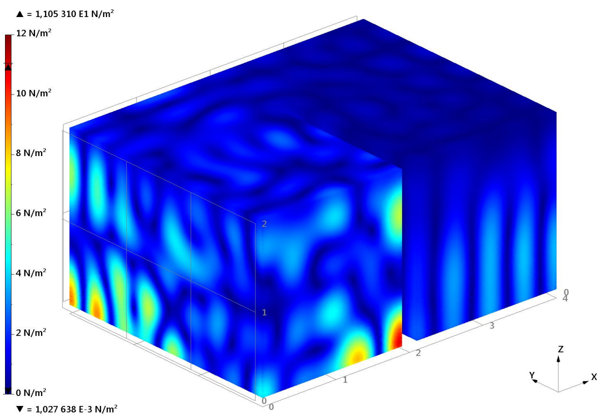

Figure 13: N_{elem} = 56,400 (BEM)
Δt_s: = 6,231 s direct (IMKL), ≈ 1:43 h
806 s iterative, GMRES, N_{iter} = 170, $e < 10^{-5}$, ≈ 0:13 h

abs(p_{surf})

Room with wall, impedance $Z=\rho c$ at ceiling, $f = 500$ Hz

MLFMM

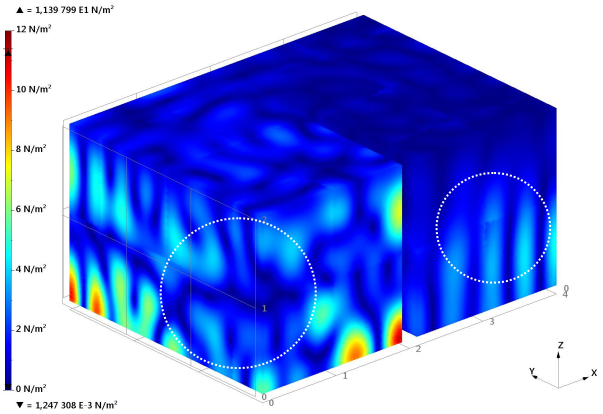

Figure 14: N_{elem} = 56,400 (BEM)
Δt_s: = 361 s iterative, MLFMM (P50/M10/U11), $N_{iter} = 163$, $e < 10^{-5}$

abs(p_{surf}) Some small but visible differences when using this multipole order (see marker)

Room with wall, impedance $Z=\rho c$ at ceiling, $f = 1$ kHz

FEM

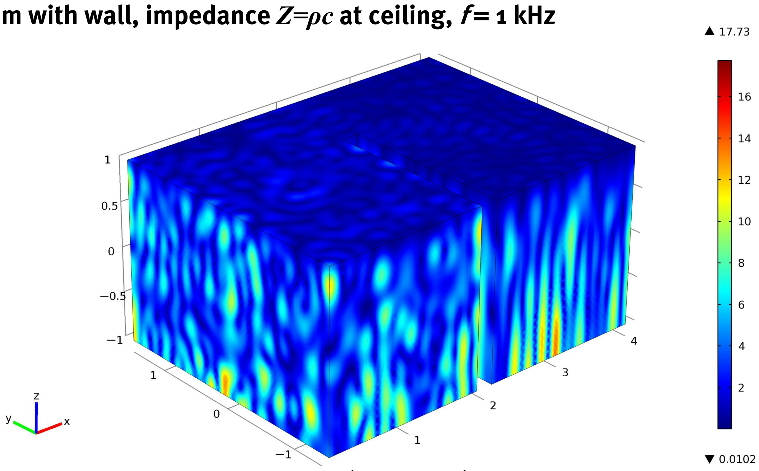

Figure 15: N_{elem} = 3,197,500 (FEM, COMSOL)
Δt_s: = 15,248 s (MUMPS), ≈ 4:14 h

abs(p_{surf})

Room with wall, impedance $Z=\rho c$ at ceiling, $f = 1$ kHz — BEM

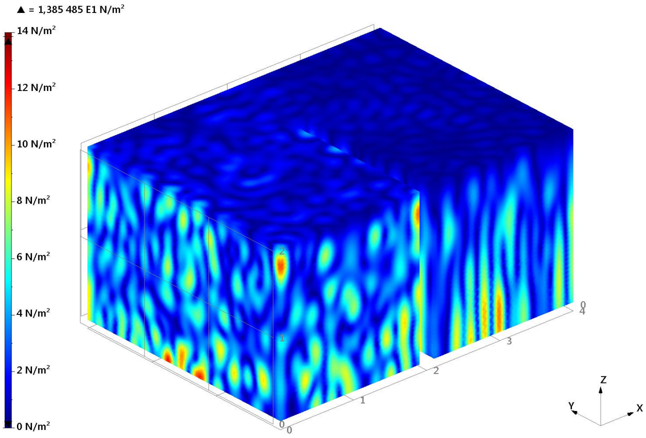

Figure 16: N_{elem} = 56,400 (BEM, $S_{mx} \approx 48.5$ GB)
Δt_s: = 6,194 s direct (IMKL), ≈ 1:43 h
2,232 s iterative, GMRES, $N_{iter} = 652$, e < 10-5, ≈ 0:37 h

abs(p_{surf})

Room with wall, impedance $Z=\rho c$ at ceiling, $f = 1$ kHz — MLFMM

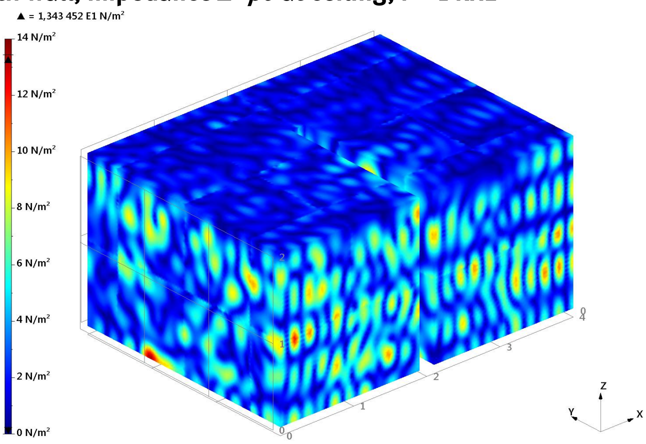

Figure 17: N_{elem} = 56,400 (BEM)
Δt_s: = 2,075 s iterative, MLFMM (P50/M14/U15), $N_{iter} = 491$, e < 10^{-5}

abs(p_{surf})

Big differences in quality although a high multipole order was used.

Test case 3: T-style room with a door-like cavity

The room is formed like an entrance area of a building with a base size of 7 × 6 × 2.5 m. The door-like cavity area, highlighted in the right figure, has an impedance of $Z = \rho c$, all other walls are rigid.

The monopole source (●) resides within the lower left part of the room at [1 1 1.25] m. The FEM mesh consists of 1,093,480 elements and the resulting surface mesh needs 30,358 elements using a maximum border length of l_{max} = 0.13 m, suitable for 400 Hz.

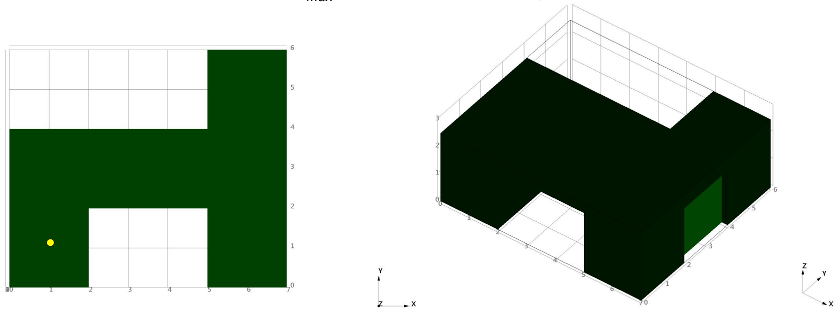

Figure 18: T-shaped room with door-like cavity

T-Room, impedance $Z=\rho c$ at "door", f= 250 Hz

FEM

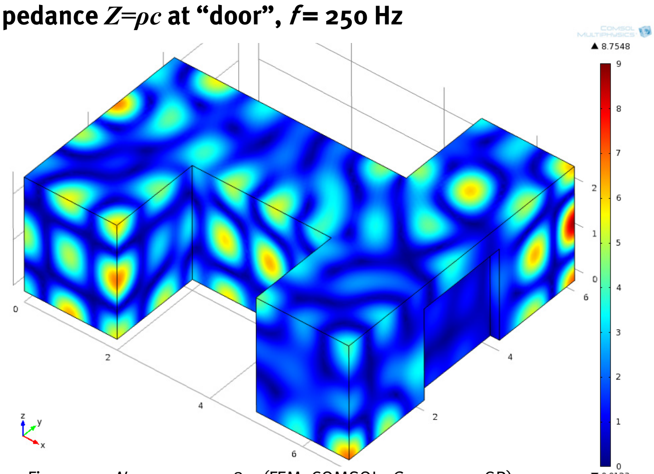

Figure 19: N_{elem} = 1.093,480 (FEM, COMSOL, $S_{mem} \approx$ 34.4 GB)
Δt_s: = 772 s (MUMPS)

abs(p_{surf})

T-Room, impedance $Z=\rho c$ at "door", f= 250 Hz **conv. BEM**

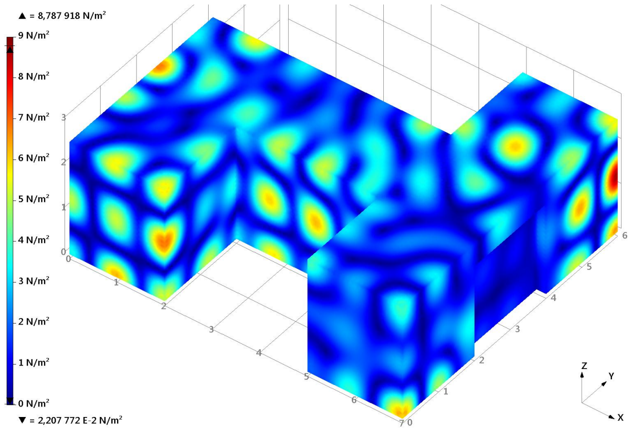

Figure 20: N_{elem} = 30,358 (BEM, $S_{mx} \approx$ 14.5 GB)
Δt_s: = 710 s direct (IMKL)
204 s iterative, GMRES, N_{iter} = 192, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z=\rho c$ at "door", f= 250 Hz **MLFMM, M6/U7**

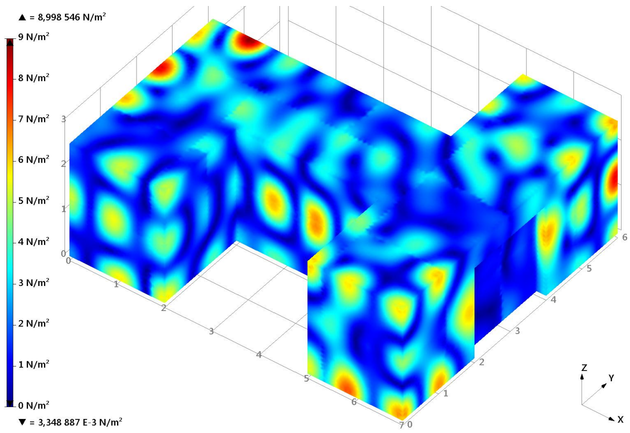

Figure 21: N_{elem} = 30,358 (BEM)
Δt_s: = 116 s iterative, MLFMM (P50/M6/U7), N_{iter} = 183, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z=\rho c$ at “door”, f= 250 Hz

MLFMM, M7/U8

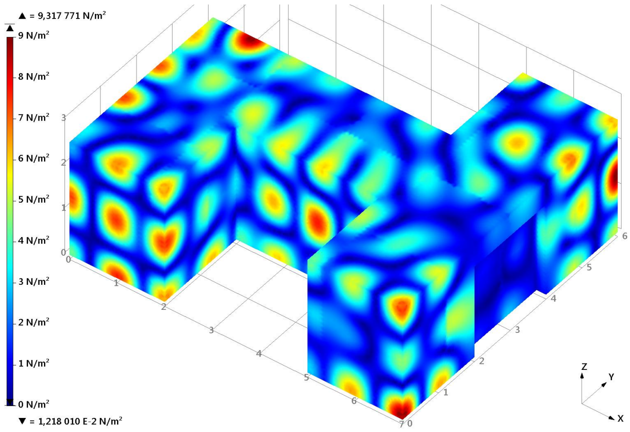

Figure 22: N_{elem} = 30,358 (BEM)
Δt_s: = 151 s iterative, MLFMM (P50/M7/U8), N_{iter} = 188, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z=\rho c$ at “door”, f= 250 Hz

MLFMM, M8/U9

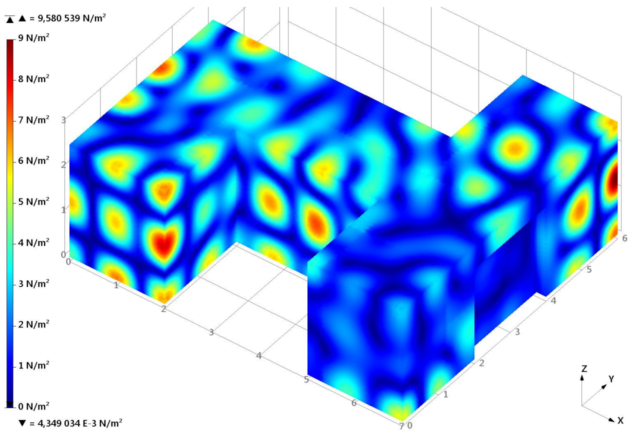

Figure 23: N_{elem} = 30,358 (BEM)
Δt_s: = 191 s iterative, MLFMM (P50/M8/U9), N_{iter} = 194, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z = \rho c$ at "door", f = 250 Hz **MLFMM, M9/U10**

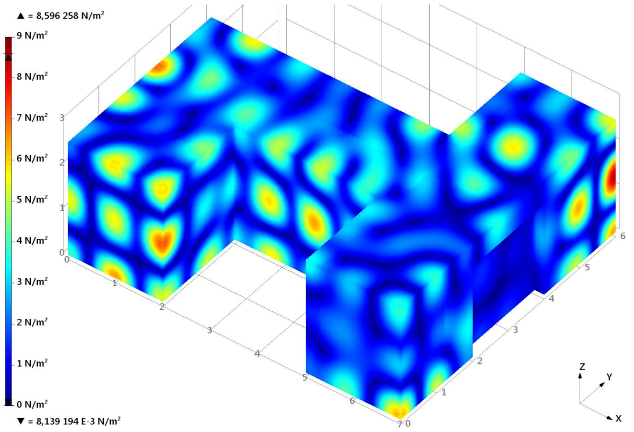

Figure 24: N_{elem} = 30,358 (BEM)
Δt_s: = 237 s iterative, MLFMM (P50/M9/U10), N_{iter} = 193, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z = \rho c$ at "door", f = 250 Hz **MLFMM, M10/U11**

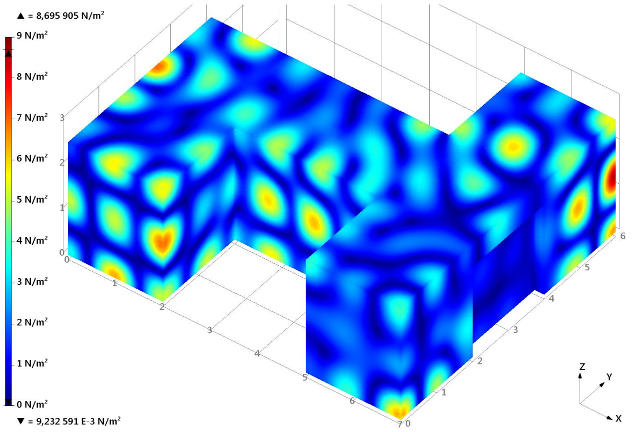

Figure 25: N_{elem} = 30,358 (BEM)
Δt_s: = 291 s iterative, MLFMM (P50/M10/U11), N_{iter} = 191, e < 10^{-5}

abs(p_{surf})

T-Room, impedance $Z=\rho c$ at “door”, $f = 250$ Hz

conv. BEM

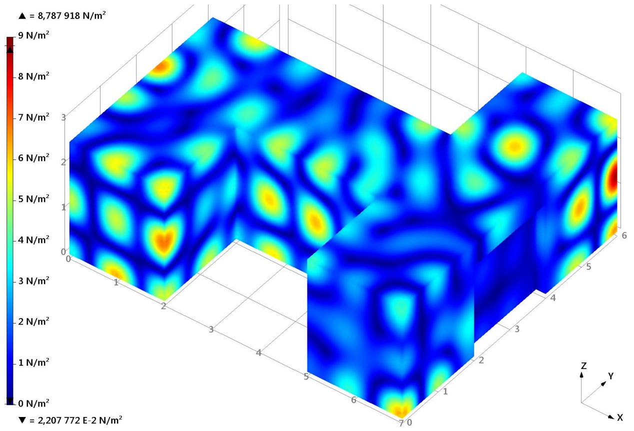

Figure 19: N_{elem} = 30,358 (BEM)
Δt_s: = 710 s direct (IMKL)
204 s iterative, GMRES, N_{iter} = 192, e < 10^{-5}

abs(p_{surf})

Conclusion

- The results achieved have demonstrated that the conventional BEM compared with the FEM method gives comparable results at lower solving times. A significant performance advantage can be achieved when treating complex structures and different kinds of boundary conditions.
- The Fast Multipole Method also seems to be applicable but has differences in quality at higher frequencies above 500 Hz especially in “quieter” regions of the structure due to the method-based errors when computing the matrix vector product and requires higher multipole orders.
- Additional investigations seem to be necessary in optimizing the MLFMM code and a level-based adaption of the multipole order.
- An adequate preconditioning of the iterative solver is needed to reduce the number of iterations because a good convergence is a precedent condition for a successful application of the MLFMM.

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Thanks for your attention! ☺

An updated version of the conference paper can be found at:

http://projekt.beuth-hochschule.de/ca

References:

[1] R. Burgschweiger, I. Schäfer and M. Ochmann: "A Multi-Level Fast Multipole Algorithm (MLFMM) for calculating the Sound scattered from Objects within Fluids", Proceedings of 20th International Congress on Acoustics, ICA 2010, Sydney, Australia

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[4] N. A. Gumerov and R. Duraiswami: "Fast Multipole Methods for the Helmholtz Equation in three dimensions", 2004, Elsevier Books, ISBN 0-08-04431-0