ACOUSTICS 2012
HONG KONG

Numerical simulation of the transmission loss of plates

Rafael Piscoya[1]*, Martin Ochmann and Ralf Burgschweiger*

Beuth Hochschule für Technik Berlin, University of Applied Sciences, Faculty II, Research Group Computational Acoustics, Luxemburger Str. 10, 13353 Berlin, Germany

Abstract: Numerical simulations for estimating the transmission loss of plates can be an important alternative to measurements when there is no access to transmission loss test facilities. Furthermore, parametric studies and design changes can be made easily and faster. This work presents a method to calculate the transmission loss of plates placed between a source and a receiver room using an iterative approach. The sound radiation due to the vibration of the plates is solved with a Boundary Element formulation while the motion of the plate is determined using a Rayleigh-Ritz formulation with the sound pressure as the exciting force. The starting point is the blocked-pressure approximation. The real pressure on the plate and its displacement are obtained after some iterations. If no damping in the plate is considered, poor or no convergence is expected at the resonant frequencies of the plate. This problem is avoided introducing some damping in the plate as well as in its fixation (boundary). With this approach, the use of existing techniques to accelerate the calculations that are already developed for the BEM, e.g. the Fast Multipole Method can be directly applied without needing to adapt them to this specific problem.
Keywords: Transmission, Boundary Element Method

1. Introduction

In acoustics, analytical descriptions are normally limited to simple systems. Therefore, measurements have been essential to determine the acoustic properties of complex systems which are usually required in practical applications. In many cases, measurements require especial environments e.g. anechoic, semi-anechoic or reverberant chambers with specific types of sound fields. The access to those especial facilities may be sometimes difficult or it may be expensive especially if the measurements have to be performed many times and for different configurations or prototypes in a design phase.

With the continuous development of computers, the complexity of the simulated systems increases. Hence, in order to reduce costs, it is desirable to perform the design optimization by means of “virtual measurements” and leave the best prototypes or the ones with the most interesting properties for the real measurements.

“Virtual transmission loss measurements” can be performed using deterministic methods like Finite Element (FEM) or Boundary Element (BEM) for low and middle frequencies. Due to the dimensions of transmission loss test facilities, the size of the discretized models can be huge for sufficient high frequencies.

In the present work, the transmission loss of thin plates is calculated by simulating a measurement in a two room test facility using BEM i.e. only the surfaces need to be discretized. An iterative approach is considered which treats each room separately. Hence, two smaller systems need to be inverted instead of a large one. Besides, each room may be solved using a Fast Multipole BEM in order to extend the range of frequencies that can be investigated.

[1] piscoya@beuth-hochschule.de

2. Problem description

The transmission loss (R) of thin plates is calculated following the procedure described in DIN EN ISO 10140-2[1]. The plate is placed between source and receiving room (see Fig. 1) and R is defined as

$$R = L_1 - L_2 + 10\log_{10}\left(\frac{S}{A}\right) \tag{1}$$

(Eq. (2) in Ref. 1), where L_1 is the mean sound pressure in source room in dB (average of pressure in field points 1), L_2 mean sound pressure in receiving room in dB (average of pressure in field points 2), S the area of the opening where the element is mounted in m^2 and A the equivalent absorbing area in the receiving room in m^2.

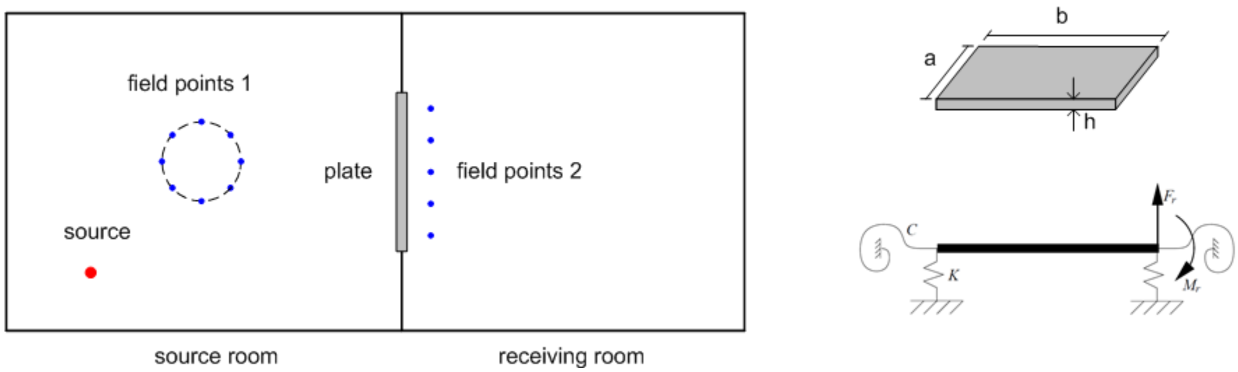

Figure 1 – Illustration of the test facility and the plate with elastic BCs

Definition (1) requires that the sound fields are diffuse and that the sound in the receiving room is exclusively due to the sound coming through the test element.

Elastic BCs can be considered by using translational and rotational springs along the boundaries with translational and rotational rigidities K_b and C_b respectively. The usual BCs are defined by the values K_b=0 and C_b=0 (free), K_b>>1 and C_b=0 (simply supported) and K_b>>1 and C_b>>1 (clamped).

3. Motion of the plate

Following the Rayleigh-Ritz variational formulation, the equation of motion of a plate is obtained by minimizing the functional:

$$H = \int_{t_0}^{t_1} \left(T - V - V_b\right) dt \tag{2}$$

where T is the kinetic energy, V the deformation energy of the plate and V_b the potential energy on the boundary. The expressions for each energy term are explicitly given in Ref. 2. The normal displacement of the plate is then expanded in a set of functions $\psi_{\mu\nu}$, $u_n = \sum_{\mu\nu} a_{\mu\nu}\psi_{\mu\nu}$. Inserting this expansion in (2), the minimization leads to the equation of motion:

$$(K - \omega^2 M)a = P \tag{3}$$

where K is the stiffness matrix, M the mass matrix, P the external force vector and a the vector of unknown coefficients.

Here, we use the set of functions suggested by Filippi[3], $\psi_{\mu\nu}(x,y) = \varphi_\mu(x)\varphi_\nu(y)$, a product of the eigenfunctions of the free beam in x and y directions. Since $\psi_{\mu\nu}$ are not bounded on the edges, they are able to satisfy arbitrary boundary conditions. The free beam functions are given by

$$\varphi_0(s)=1 \quad , \quad \varphi_1(s)=s-L_s/2 \quad ,$$
$$\varphi_m(s)=\sinh\delta_m s+\sin\delta_m s-\kappa(\delta_m L_s)(\cosh\delta_m s+\cos\delta_m s)\,, \quad m=2,3,... \tag{4}$$

where $\kappa(\beta)$ = $(\cosh\beta - \cos\beta)$ / $(\sinh\beta + \sin\beta)$ and $\delta_m L_s$ are solutions of the transcendent equation $\cosh\delta_m L_s \cos\delta_m L_s = 1$.

4. Sound field in the rooms

The sound pressure in the rooms is calculated using the BEM. The whole surface of the acoustic domain is subdivided in 4 surfaces, S_1-S_4 with normal vectors n_1-n_4 as shown in Fig. 2. S_2 correspond to the plate and S_3 to the rigid surface between the rooms.

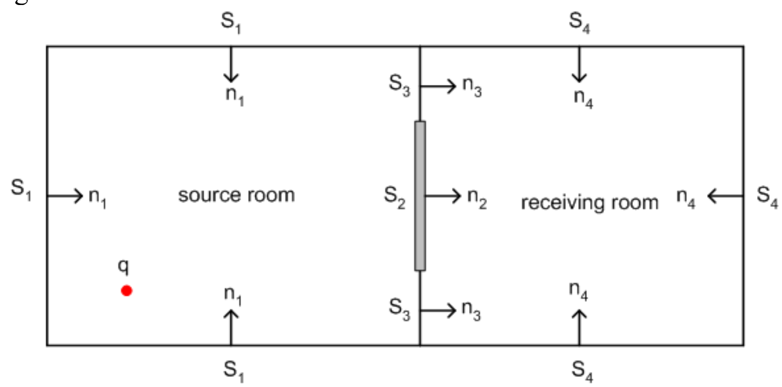

Figure 2 – Illustration of the test facility and the plate with elastic BCs

The integral equations for the rooms are given by

$$C_I p_I = \int_{S_1}(p_I\frac{\partial g}{\partial n}-\frac{\partial p_I}{\partial n}g)dS - \int_{S_2}(p_I\frac{\partial g}{\partial n}-\frac{\partial p_I}{\partial n}g)dS - \int_{S_3} p_I\frac{\partial g}{\partial n}dS + p_q \tag{5}$$

$$C_{II} p_{II} = \int_{S_2}(p_{II}\frac{\partial g}{\partial n}-\frac{\partial p_{II}}{\partial n}g)dS + \int_{S_3} p_{II}\frac{\partial g}{\partial n}dS + \int_{S_4}(p_{II}\frac{\partial g}{\partial n}-\frac{\partial p_{II}}{\partial n}g)dS \tag{6}$$

where p_q is the sound pressure due to the sound source q.

In the present work, we are considering only thin structures. Therefore, $\partial p_I/\partial n = \partial p_{II}/\partial n = \rho\omega^2 u_n$ on S_2. S_1 and S_4 are considered absorbing surfaces in order to decrease the amplitude of the room resonances. For those surfaces $\partial p/\partial n = j\rho\omega p/Z$ holds. Discretization of Eqs. (5) and (6) on the boundary leads to two matrix equations

$$A_I p_I^S = G_I\rho\omega^2 u_z + p_q \tag{7}$$

$$A_{II} p_{II}^S = -G_{II}\rho\omega^2 u_z \tag{8}$$

that provide the values of the sound pressure on the whole surface assuming u_z is known. Here $p_I^S = \begin{bmatrix} p_{I1} & p_{I2} & p_{I3} \end{bmatrix}^T$ and $p_{II}^S = \begin{bmatrix} p_{II2} & p_{II3} & p_{II4} \end{bmatrix}^T$.

5. Iteration scheme

The motion of the plate and the sound field in the rooms are coupled through the force vector P in Eq. (3)

$$P_{nm} = \int_{S_2} \psi_{nm}(p_I - p_{II})dS \tag{9}$$

The iteration starts assuming that the plate does not vibrate $u_z^{(0)} = 0$ (blocked-pressure approximation). The sound pressures at both sides of the plate ($p_{I2}^{(0)}$ and $p_{II2}^{(0)}$) are calculated using Eqs. (7) and (8). With those known pressures, the force vector can be determined using (9) and the next value of the displacement can be computed. The calculations are repeated until the difference between step n-1 and n is smaller than a certain value or the maximum number of iterations is reached. The n-th double step of the procedure can be written as

$$\Delta p_M^{(n)} = (L_f^I + L_f^{II})u_z^{(n-1)} \quad , \quad u_z^{(n)} = L_p(\Delta p_M^{(n)} + \Delta p_E) \tag{10}$$

In Eq. (10), Δp_M is the pressure difference due to the motion of the plate and Δp_E is the pressure difference due to the sound sources. L_p is the operator describing the motion of the plate and L_f^I and L_f^{II} are the operators regarding the excitation of the plate due to the sound pressure.

The iterative approach will be useful if the solution converges to the right solution with a small number of iterations. Introducing some damping to the plate and to its elastic boundary conditions as well as absorption in the walls of both rooms, the number of iterations should decrease.

A combination of Eqs. (10) provides a single step iteration of the form

$$u_z^{(n)} = T\, u_z^{(n-1)} + \phi \tag{11}$$

The iteration will converge if the spectral radius of the system matrix T is less than 1. The spectral radius is defined as $max(|\lambda_i|)$, where λ_i are the eigenvalues of T.

6. Numerical results

For the evaluation of the iterative approach, the window test facility of the "Institut für Bauphysik – Fraunhofer Institut" was considered. The source and receiving rooms are rectangular rooms with dimensions 5.74m × 3.75m × 3.11m and 4.85m × 3.75m × 3.11m respectively. The opening is also rectangular with dimensions 1.25m × 1.5m. For simplicity, the width of the wall dividing the two rooms was neglected. In the source room a small absorption was considered $\alpha = 0.18$ while in the receiving room a high absorption $\alpha = 0.89$ was used. The first ten resonances of the source and receiving rooms, assuming rigid walls, are listed in Table 1. Since absorption is considered, the true resonances will be slightly shifted.

A point source was placed near one corner of the source room. Field points 1 are placed on a sphere of radius 0.75m centered approximately on the middle of the source room. Field points 2 were set 0.5m away from the plate (see Figure 1).

The simulated plate is an aluminum plate with the dimensions of the opening and a thickness of 0.004m. An elastic BC was assumed for the plate ($K_b = 1\cdot10^7$ and $C_b = 7\cdot10^2$). For aluminum, the values taken for Young's modulus, density and Poisson's ratio are: $E = 64\cdot10^9$ Pa, $\rho = 2{,}700$ kg/m^3 and $\nu = 0.3$. Damping was introduced by defining complex Young's modulus $E(1+j\eta_E)$ with a $\eta_E = 0.05$ and complex stiffness $K_b(1+j\eta_b)$ and $C_b(1+j\eta_b)$ with $\eta_b = 0.1$. The first ten resonance frequencies of the plate, computed using the basis of beam functions, are also listed in Table 1.

Table 1 – Resonance frequencies

Resonances	*source room*	*receiving room*	*plate*
1	29.9	35.6	12.1
2	45.7	45.7	24.4
3	54.6	55.1	30.1
4	55.1	57.8	42.2
5	59.8	65.5	44.9
6	62.7	70.7	59.6
7	71.6	71.6	62.5
8	75.2	79.9	71.6
9	77.6	84.2	73.5
10	81.3	89.7	90.9

The discretized model is made of about 24,000 rectangular elements. The size of the elements ensures accuracy of the results up to 500 Hz. The calculations were made up to 800 Hz.

Figure 3 shows the results of the numerical simulation. The transmission loss obtained with the iterative approach is illustrated together with the results of a direct calculation on the top left plot. In the direct calculation, the coupled system of equations is solved. Both curves are practically identical, because the differences are very small, below 0.3 dB as shown in the bottom left plot. On the right side, one can see on the top the spectral radius of the system matrix T of Eq. (11) and on the bottom the number of iterations needed. Since the spectral radius is smaller than 1 for all frequencies, convergence of the iterative method is ensured. The number of iterations is higher for values of the spectral radius near 1 than for lower values as expected.

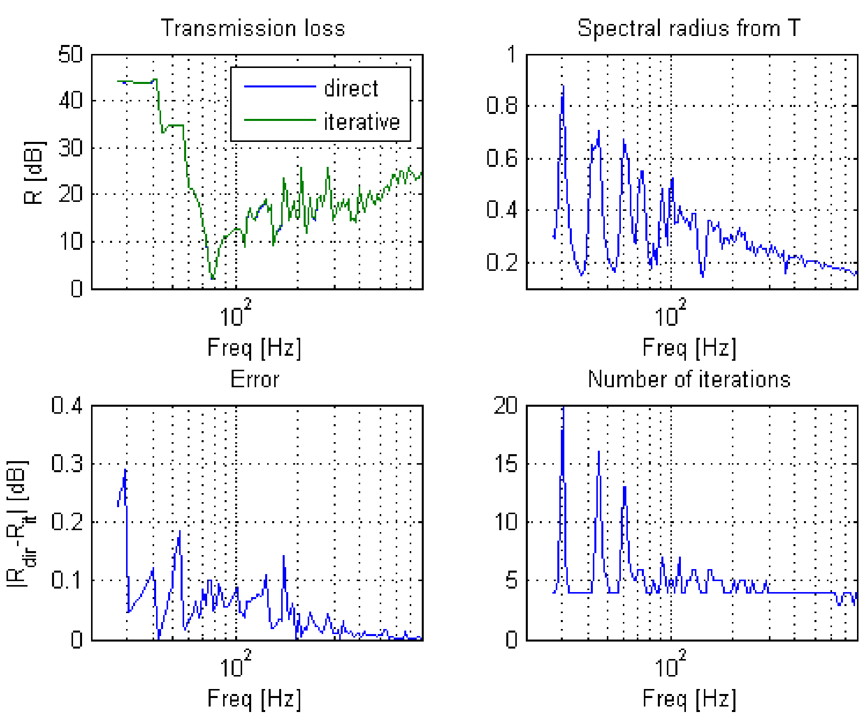

Figure 3 – Transmission loss and other parameters

Keeping the rooms the same and varying only the thickness of the plate for the same BCs, it was observed that thicker plates had lower values of the spectral radius while thinner plates bigger values, including values greater than 1. Apparently, the damping on the plate should be increased for decreasing thickness of the plate to ensure convergence of the iterative method.

Figure 4 shows the sound pressure level in both rooms for three different frequencies, 60 Hz, 79 Hz and 150 Hz and the corresponding displacement of the plate. The displacement is shown in dB respect to 10^{-9}m. The minimum in the transmission loss curve appears at 79 Hz.

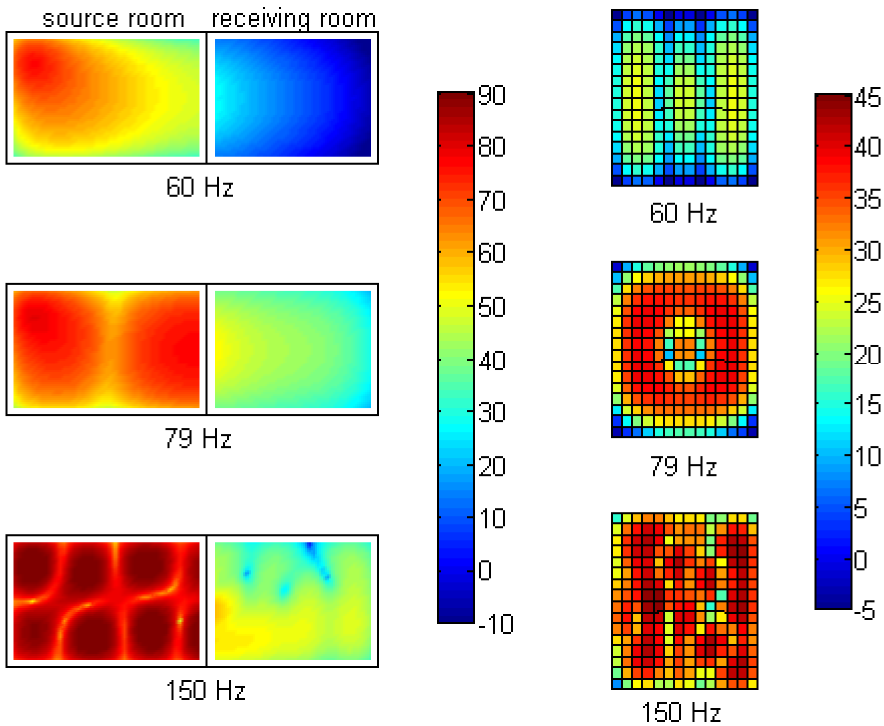

Figure 4 – Sound pressure distribution (left) and normal displacement of the plate (right)

Acknowledgements

This work was supported by the German Research Foundation (DFG) within the project "General computational model for sound transmission problems".

References and links

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