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THE USE OF AN ALTERNATIVE BEM ANSATZ FUNCTION FOR SCATTERING PROBLEMS

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The frequency dependent backscattering of an obstacle can be calculated by means of different numerical methods. The use of classical methods like the conventional BEM or FEM leads to high computing times in combination with large memory requirements due to the required discretization efforts in the higher frequency range.
Classical high-frequency approximation procedures like the Kirchhoff- or the Plane-Wave-method reduce the calculation time at the expense of imprecise solutions.
This paper will present the results of a new approach which combines the advantages of all the above mentioned methods using alternative ansatz functions. This approach allows it to use a discretization which needs just one element per wavelength. So in comparison with the common rule of thumb of six elements per wavelength even much larger problems and/or higher frequencies can be calculated.

1. Introduction

To calculate the frequency-dependent acoustic backscattering intensity of an underwater object, various methods can be used. First, classical methods (FEM / BEM) may be used but they lead to large computation times and memory requirements due to the fine element mesh descretization required at high frequencies. High-frequency approximation methods (Kirchhoff, PWA) may also be used which need low computation time but produce less accurate solutions.

To decide whether a high-frequency approximation method can be used in this paper a new approach for the BEM function is derived and tested, which reduces the computational requirements of the classical BEM by reducing the required elements significantly. The results are compared in terms of quality and performance. All calculations were performed using a special application to the BEM-based simulation of scattering problems [1].

2. Acoustic backscattering intensity (target strength, TS)

The acoustic backscattering intensity or target strength (TS) is defined as the ratio of incident to reflected sound intensity [2]. In order to obtain comparable values, a sound source is placed in the far field of the object and then the backscattering intensity is calculated back to a distance of one meter to the object determined. It satisfies the following formula:

$$TS = 20\lg\left(\frac{p_{scat}}{p_{inc}}\right). \tag{1}$$

Here, p_{scat} is the pressure amplitude of the signal reflected from the object in the far field and p_{inc} the corresponding one of the incident wave. At a given pressure and speed on the boundary Γ of the known object, the backscattered pressure by means of the Kirchhoff integral can be calculated by

$$p_{scat} = \int_\Gamma \left[g\frac{\partial p}{\partial n} - p\frac{\partial g}{\partial n}\right] \mathrm{d}\Gamma \tag{2}$$

with g being the fundamental solution of the Helmholtz equation.

3. Kirchhoff (KIA) and plane wave approximation (PWA)

The above-mentioned integral (2) requires knowledge of the pressure and the velocity on the surface of the object. A relationship between p_{scat} and p_{inc} by means of a reflection coefficient can be defined depending on the acoustic direction:

$$p_{scat} = Rp_{inc}, \quad \frac{\partial p_{scat}}{\partial n} = -R\frac{\partial p_{inc}}{\partial n}, \quad p = p_{scat} + p_{inc}\,. \tag{3}$$

This coefficient is the ratio of pressure and normal velocity of the incident to the scattered sound wave directly on the surface and can be calculated using the specified Brekhovskikh [3] process. The approximation is based on the assumption that the law of reflection for plane waves and infinite plates can be applied on each element. For this reason it is particularly true for high frequencies and only for convex objects. In this paper, only the sound-hard case ($R = 1$) which is relevant for very high frequencies is considered.

Using the Kirchhoff high-frequency approximation [4] for the illuminated part $\Gamma_{ill.}$of the surface holds

$$p_{scat} = -2\int_{\Gamma_{ill.}} p_{inc}\frac{\partial g}{\partial n}\,\mathrm{d}\Gamma \tag{4}$$

while for the PWA high-frequency approximation [4], the integration over the entire surface must be performed:

$$p_{scat} = -\int_\Gamma \left(1-\cos\phi\right) p_{inc}\frac{\partial g}{\partial n}\,\mathrm{d}\Gamma\,. \tag{5}$$

The angle ϕ is build from the direction of the incident wave $\vec{n}_{inc}$ and the element normal $\vec{n}$ of the respective element y as shown in Fig. 1.

Both Eqs. (4) and (5) now have the advantage that all the required quantities are known and thus the integral can be calculated.

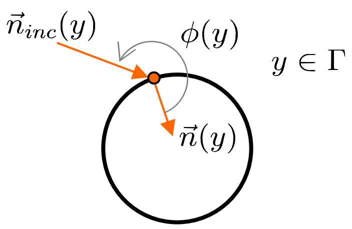

Fig. 1. normal vectors used in the PWA

As an example, the target strength of a cylinder with spherical caps consisting of about 76,000 elements, which is irradiated under an aspect angle of 30°, is calculated for 20 kHz. In Fig. 2, the real part of the sound pressure on the surface is shown for the Kirchhoff method, in Fig. 3 for the PWA and in Fig. 4 for the conventional BEM.

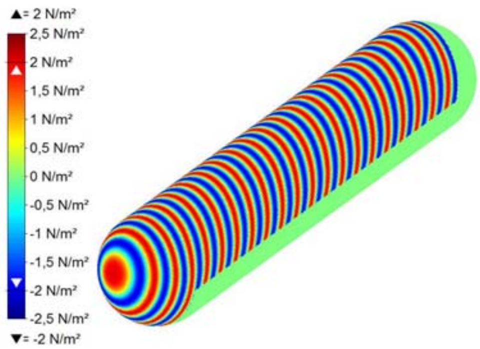

Fig. 2. Kirchhoff approximation, $\mathrm{Re}(p_{surf})$

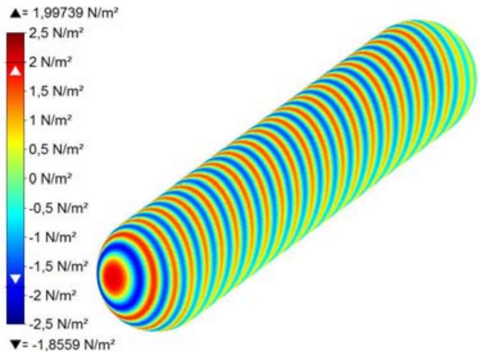

Fig. 3. Plane wave approximation, $\mathrm{Re}(p_{surf})$

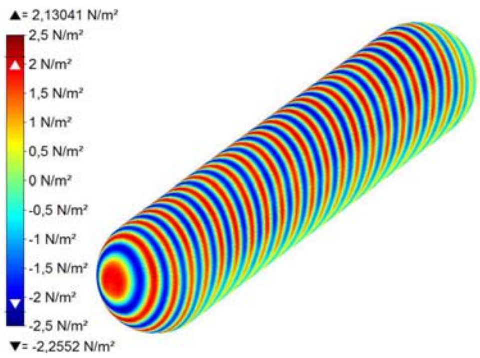

Fig. 4. conventional BEM, $\mathrm{Re}(p_{surf})$

The resulting target strength (Fig. 5) is all about the same level.

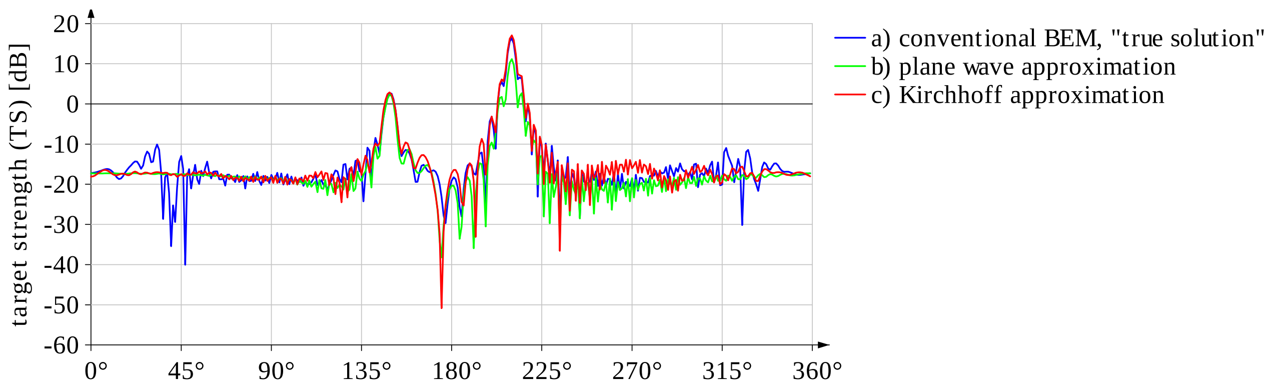

Fig. 5. target strength of a rigid cylinder at f = 20 kHz

But in certain cases, the results of the two high-frequency method, however, do not match well. Another cylinder with one flat end which is hit from two different aspect angles serves as an example using a frequency range from 0.1 to 10 kHz.

In Fig. 6, the angle of incidence was 90° and the target point was at an angle of 180°. Here, the PWA (green) is in good agreement with the "true solution" (blue).

In Fig. 7, the target point was at an angle of 150° and the incident angle was 30°. In this case, Kirchhoff (red) matches with the "true solution" (blue).

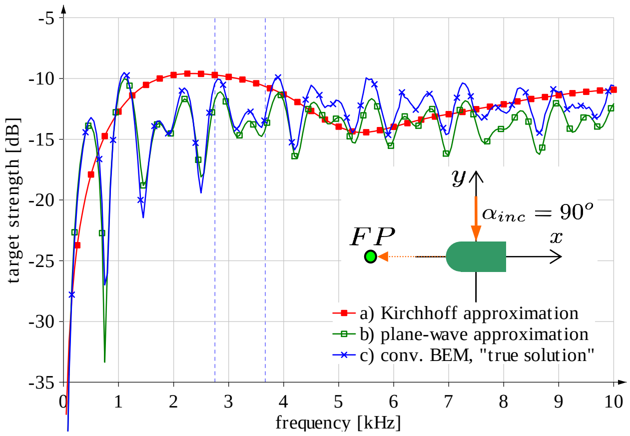

Fig. 6. Target strength for α_{inc} = 90°, α_{fp} = 180°

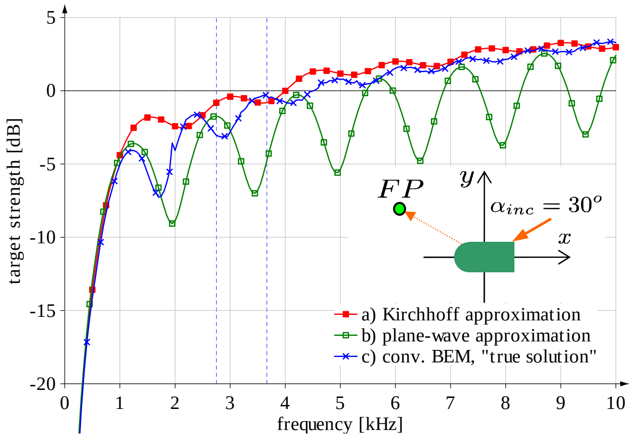

Fig. 7. Target strength for α_{inc} = 30°, α_{fp} = 150°

The reason for this lies in the prefactors of the Eq.s (4) and (5). For the PWA this prefactor, $(1 - \cos\phi)$, depends on the element normal and takes values between 0 and 2. For the Kirchhoff approximation, this value is 2 on the illuminated side and vanishes on the "dark" side.

4. BEM formulation with a plane-wave based ansatz function

To determine this prefactor precisely, the following approach is chosen for the pressure at any point $\vec{r}$:

$$p\,(\vec{r}) = A(\vec{r})\,p_{inc}(\vec{r})\,. \tag{6}$$

The relevant formula for the classical BEM can be obtained by means of the Kirchhoff integral (2). It reads in discretized form (using triangles) for the rigid case:

$$\frac{1}{2}p(\vec{y}) = -\sum_{N_{elem}} \iint_{\Delta} p(\vec{x})\,\frac{\partial g(\vec{x},\vec{y})}{\partial \vec{n}_x}\,\mathrm{d}\Gamma + p_{inc}(\vec{y}) \tag{7}$$

with $\vec{x}$ Variable of integration, "field point"
$\vec{y}$ load point
$\vec{n}$ normal vector at field point

To solve the integrals over the triangles in (7), they must be made so small that the pressure can be set outside the integral when using constant shape functions. Using the approach in Eq. (6) for $\vec{x}$ and $\vec{y}$ with Eq. (7) holds

$$\frac{A(\vec{y})}{2} = -\sum_{N_{elem}} \iint_{\Delta} A(\vec{x})\,\frac{p_{inc}(\vec{x})}{p_{inc}(\vec{y})}\,\frac{\partial g(\vec{x},\vec{y})}{\partial \vec{n}_x}\,\mathrm{d}\Gamma + 1\,. \tag{8}$$

Since the prefactor $A(\cdot)$ can be assumed to be constant for much larger areas, it can also be taken outside the integral $\iint_{\Delta}$. The resulting system of equations no longer determines the unknown pressure p as in the classical BEM, but the unknown factor A. This system of equations now has far fewer unknowns than when using the classical constant or linear shape functions.

4.1 Example 1: small cylinder

Here, the pressure on the surface of a small cylinder (2 × 1 × 1 m) with one round cap is calculated. The cylinder is irradiated from 30° by a monopole source with a frequency of f = 10 kHz.

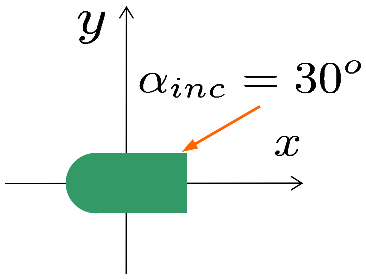

Fig. 8. small rigid cylinder

For the two left results (Fig. 9 and Fig. 11), constant shape functions were used while the right solutions (Fig. 10 and Fig. 11) were calculated with the plane-wave ansatz function. The top cylinders consist of 2,500 elements, the bottom ones of 32,000 elements.

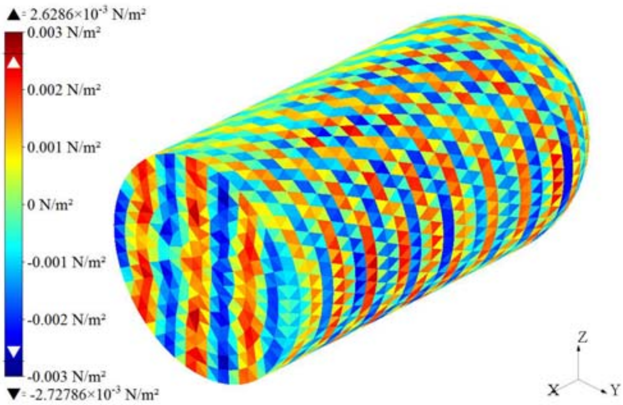

Fig. 9. $\mathrm{Re}(p_{surf})$, N_{elem} = 2,500
Δt_{solve} = 6.89 s

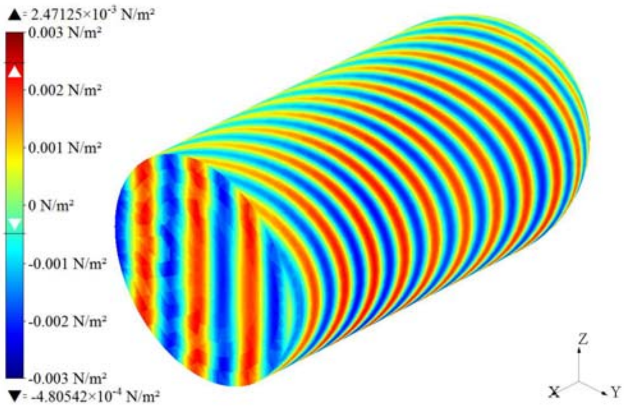

Fig. 10. w/PWA, $\mathrm{Re}(p_{surf})$, N_{elem} = 2,500
Δt_{solve} = 9.38 s

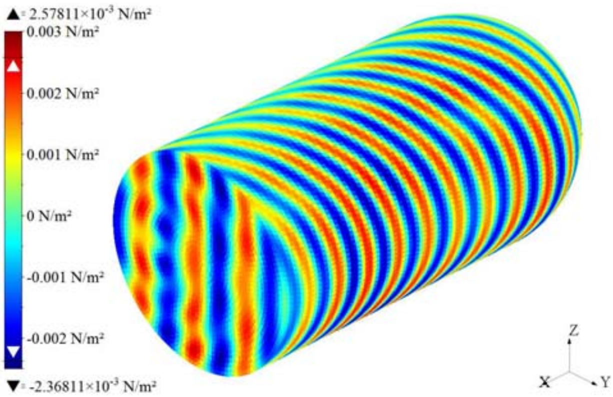

Fig. 11. $\mathrm{Re}(p_{surf})$, N_{elem} = 32,000
Δt_{solve} = 1,351.8 s

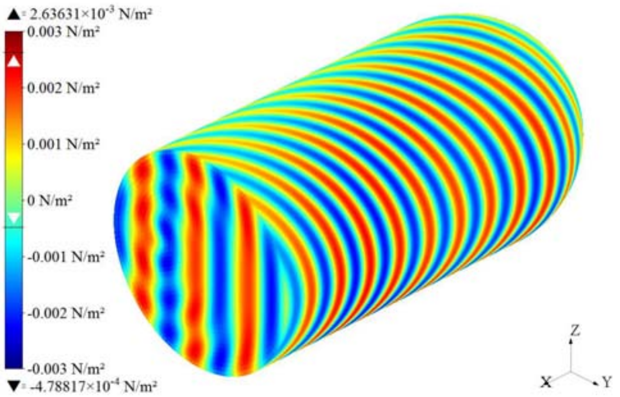

Fig. 12. w/PWA, $\mathrm{Re}(p_{surf})$, N_{elem} = 32,000
Δt_{solve} = 1,423.4 s

Although only 2,500 elements (Fig. 10, top right) are used, the result is almost as good as the 32,000 elements solution (Fig. 12, bottom right).

Calculating the target strength results in a similar situation. The classical constant or linear shape functions require about 6 elements per wavelength. In contrast, using the plane wave ansatz function requires only one element per wavelength (for convex surface portions).

4.2 Example 2: submarine model

The sound pressure on the surface of a submarine model (overall size: 57 × 14 × 13 m, $l_{k,max}$ = 0.95 m, f = 1.5 kHz, λ = 1 m, N_{elem} = 21,062) in water was calculated without (Fig. 13) and with the additional ansatz function (Fig. 14). All figures will show the real part of the pressure on the surface and all colours are adjusted to represent the same range.

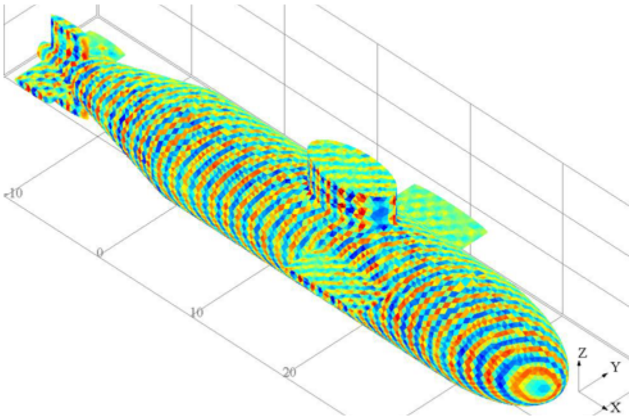

Fig. 13. $\mathrm{Re}(p_{surf})$, conventional BEM, Δt_{solve} = 303 s (direct solver, Intel MKL)

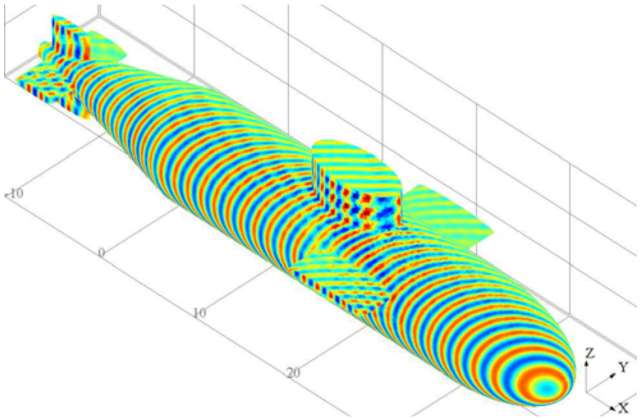

Fig. 14. $\mathrm{Re}(p_{surf})$, with ansatz function, Δt_{solve} = 389 s (direct solver, Intel MKL)

The quality of the conventional BEM is worse due to the "large" elements (with regard to the wavelength). The quality is much higher when using the ansatz function, only within the areas with multiple reflection the progression of sound pressure is worse.

5. Using the ansatz function to determine reflecting surfaces

The irregularities can be seen clearly using the submarine model of chap. 4.2 with much smaller elements ($l_{k,max}$ = 0.23 m, N_{elem} = 175.412, Fig. 15) and looking at the tower and the fins of the coarser mesh (Fig. 16).

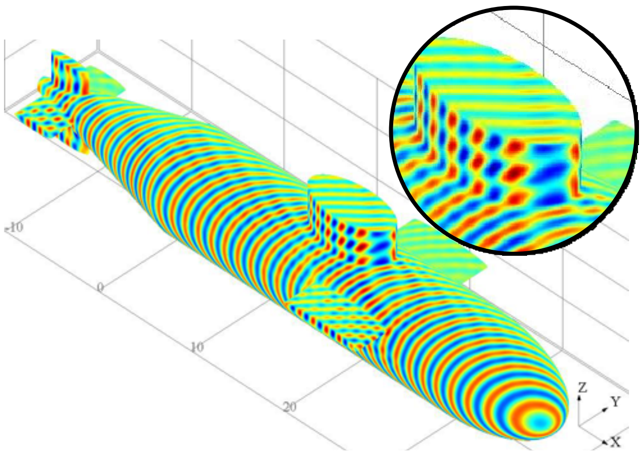

Fig. 15. $\mathrm{Re}(p_{surf})$**,** conventional BEM, f = 1.5 kHz, N_{elem} = 175,412 Δt_{solve} = 23,255 s [6:27:35] with GMRES

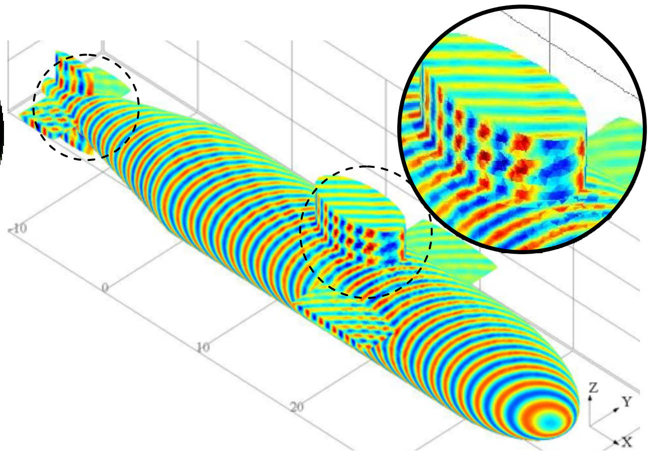

Fig. 16. $\mathrm{Re}(p_{surf})$, with ansatz function, f = 1.5 kHz, N_{elem} = 21,062 Δt_{solve} = 389 s (direct solver, Intel MKL)

So the additional function may be used for the identification of surface areas where multiple reflections occur (concave areas).

To get these areas of interest, the ratio between the imaginary and the real part of the prefactor A is used (Fig. 17). The use of a threshold function removes the areas with small surface pressure values (Fig. 18).

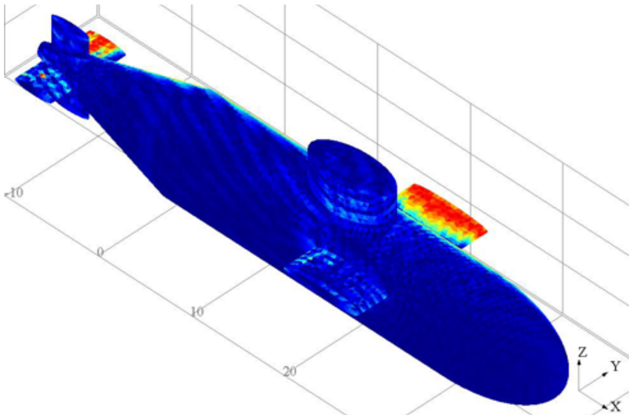

Fig. 17. ratio between $\mathrm{Im}(p_{surf})$ and $\mathrm{Re}(p_{surf})$

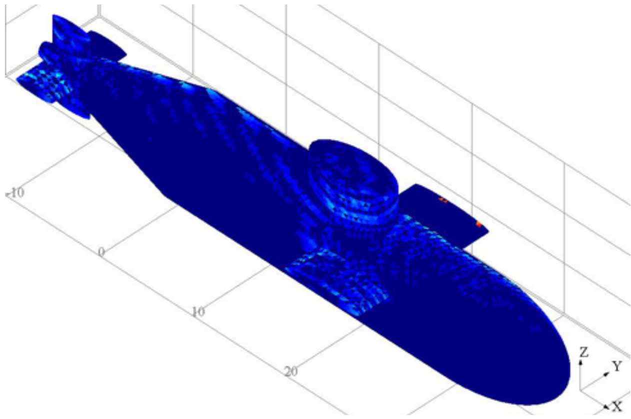

Fig. 18. applying an threshold function for “small” pressure values

Applying a geometric filter to select and mark all elements which are part of a multi-reflecting surface area (Fig. 19) makes it possible to refine these areas in a new surface mesh (Fig. 20).

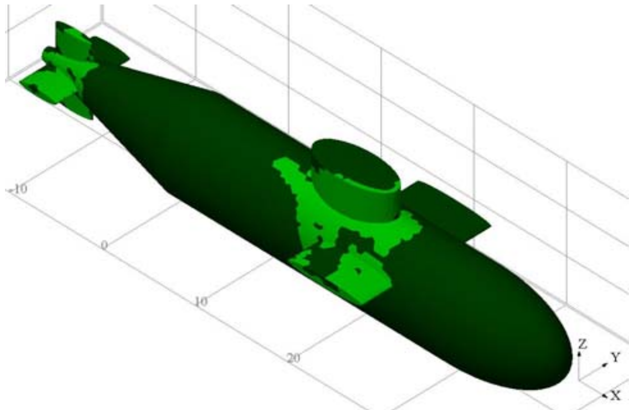

Fig. 19. marked surface areas to refine

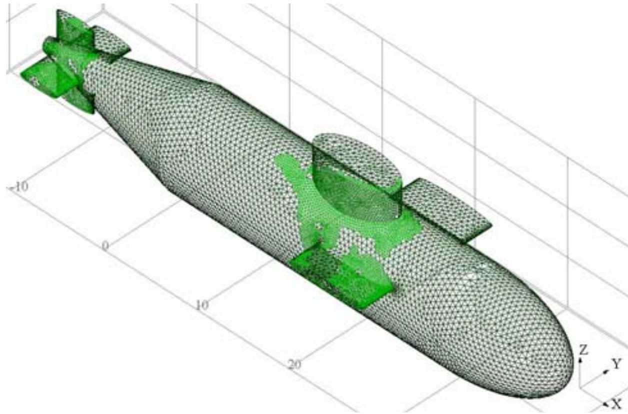

Fig. 20. refined areas, N_{elem} = 31,622

The results of the partially refined mesh are good (Fig. 21) and very near compared to the finest mesh (Fig. 15).

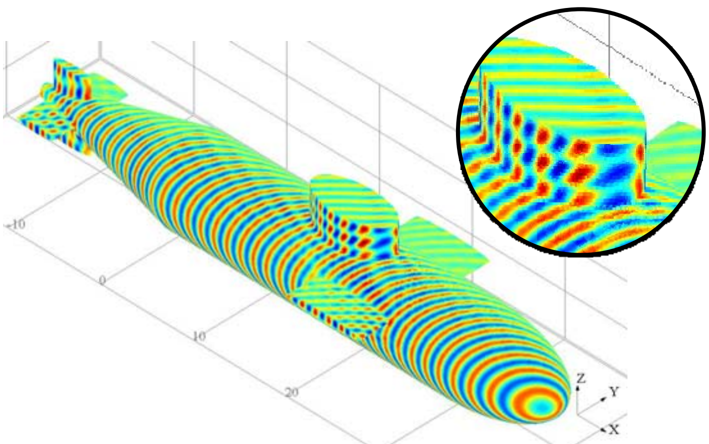

Fig. 21. $\mathrm{Re}(p_{surf})$, with ansatz function and refined areas,
f = 1.5 kHz, N_{elem} = 31,622
Δt_{solve} = 1,112 s [0:18:31] (direct solver, Intel MKL)

The results for the different mesh sizes and methods can also be seen when comparing the real part of the surface pressure at the lower side of the front fin (Figs. 22 ... 25).

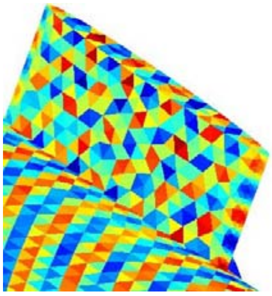

Fig. 22.
N_{elem} = 21,062
conventional BEM

Δt_{solve} = 303 s

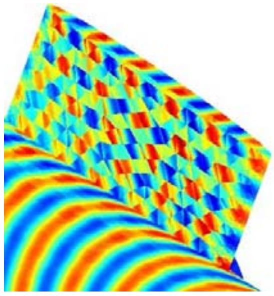

Fig. 23.
N_{elem} = 21,062
with ansatz function

Δt_{solve} = 389 s

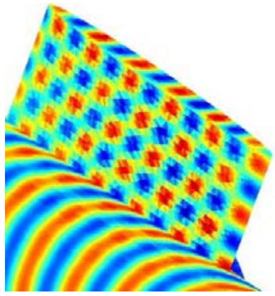

Fig. 24.
N_{elem} = 31,622
refined mesh
with ansatz function
Δt_{solve} = 1,112 s
[0:18:31]

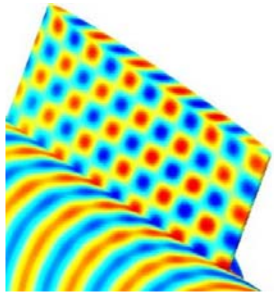

Fig. 25.
N_{elem} = 175,412
conventional BEM
with GMRES
Δt_{solve} = 23,255 s
[6:27:35]

The use of the partially refined mesh gives acceptable results at a reasonable solution time. At this time, the ansatz function works fine when using a direct matrix-based solver (Intel Math Kernel Library) but did not converge well with an iterative solver (GMRES). So the condition of the system matrix should be optimized, e.g. by adding the Burton-Miller-formulation to the ansatz function.

6. Conclusions and future work

The use of the new plane-wave based ansatz function for structures with primarily convex surfaces gives very good results and allows the use of coarse meshes with about one element per wavelength, while classical constant or linear shape functions require about 6 elements per wavelength. This allows much larger problems to be solved using the same hardware.

If this method is used for the detection of multiple reflective surface portions, this problem can be solved with significant performance advantages by a partial refinement of the mesh.

In addition to the extension of the method to the Burton-Miller approach, further work on the combination of this method with other approximate methods has to be done.

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