The Combination of a Multi-Level Fast Multipole Algorithm with a Source-Clustering Method for higher expansion orders

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Introduction

The Multi-Level Fast Multipole Method (MLFMM) allows the computation of acoustical problems based on the Boundary Element Method (BEM) where the discretized models may consist of a huge number of elements.

The required multipole expansion order of the so-called translation operator increases significantly with higher frequencies and larger dimensions of the model considered.

This problem can be avoided by combining the MLFMM with a Source Clustering Method (SCM) that replaces this translation operator by a summation method, which considers the interactions between the relevant points of the source and target clusters.

Multi-Level Fast Multipole Method

The Multi-Level Fast Multipole Method (MLFMM) describes a fast algorithm to accelerate the matrix-vector product which is required for the iterative solution of BEM-based calculations without ever assembling the complete matrix. Details of the fundamentals of the MLFMM can be found in [1, 2, 3, 4], details of the practical implementation of the code and results can be found in [5, 6, 7].

One of the most critical functions with respect to the numerics is the translation operator μ^M. This operator "transfers" the multipole potential from a source cluster ("source box") to the center of a target cluster ("target box") in the so-called "Multipole-to-Local"-step (M2L) and can be represented as a truncated series [Eq. (1)].

$$\mu^M(\vec{a}, \hat{s}) = \frac{1}{4\pi} \sum_{l=1}^{O_{mp}} (2l+1)\, i^l\, h_l(k\,|\vec{a}|)\, P_l(\hat{s}\cdot\hat{a}) \qquad (1)$$

with
$\vec{a}$ distance vector between cluster centers
$\hat{a}$ normalized distance vector
$\hat{s}$ set of vectors on the unit sphere
O_{mp} multipole order (maximum)
k wave number
$h_l(k\,|\vec{a}|)$ Hankel function
$P_l(\hat{s}\cdot\hat{a})$ Legendre polynomials

The value required for the multipole order depends primarily on the wave number and the cluster distance. If a value of 80 - 90 is exceeded, one can hardly achieve reasonable results for the series due to numerical inaccuracies of the Hankel functions and high memory requirements for the coefficients of the unit sphere, which is used for integration.

A worst case is e.g. having two smaller structures with a bigger distance between them (Figure 1).
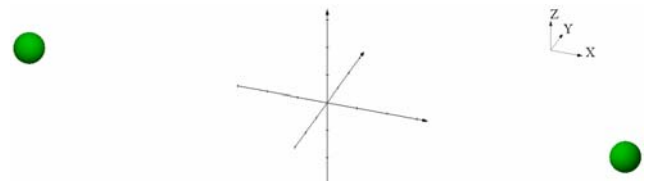

Figure 1: Two small spheres with a big distance in between

Because of the distance, the surrounding "main" box is very large (Figure 2), requiring very high values for the multipole order O_{mp}, but the interaction between these structures is not really significant.
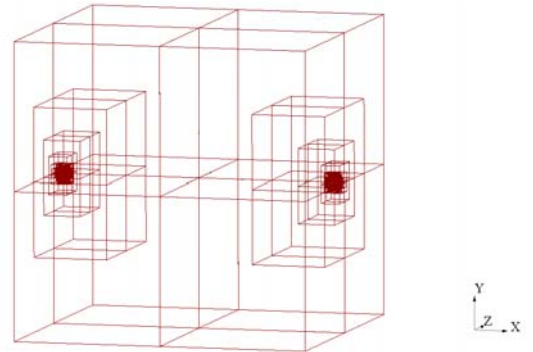

Figure 2: Cubic clusters around two small spheres

Example: Cylinder with round end cap at 10 kHz

A cylinder with one rounded end cap is hit by a plane wave at an incident angle of 30° and used to illustrate the problem (dimensions: $3 \times 1 \times 1$ m, element size $l_{max} = 0.027$ m, $N_{elem} = 57{,}652$ triangular constant elements, Figure 3).
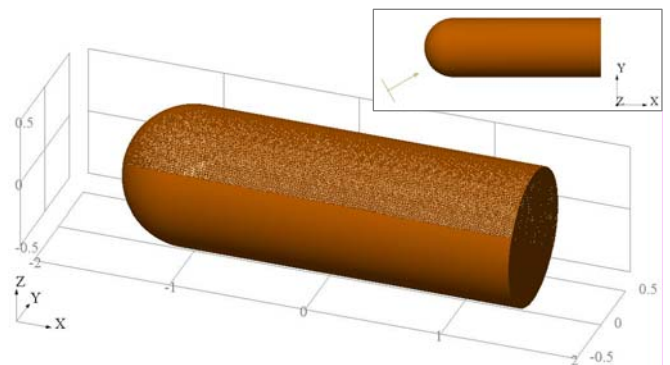

Figure 3: Rounded cylinder consisting of 57,652 elements

The 3:1 ratio between the diameter and the length results in large distances between the clusters. After applying the "cluster boxing" algorithm, a set of 6,328 cubic boxes at 7 levels is generated (Figure 4).
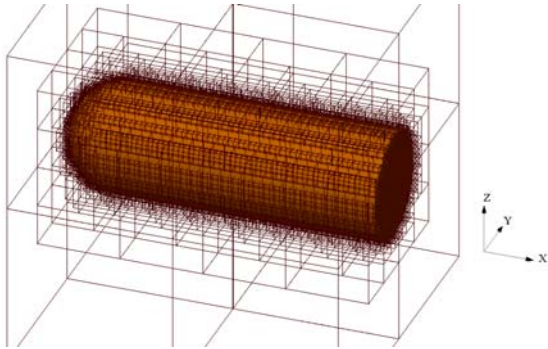

Figure 4: Cubic cluster boxes around the cylinder (maximum number of points per box $N_{p/box}$ = 30)

At a test frequency of f = 10 kHz the resulting wave number is $k \approx 41,89$ 1/m (water, density ϱ = 1,000 kg/m³, sound speed c = 1,500 m/s, wave length λ = 0.150 m). This wave number and the distances between the clusters on the same level are used to calculate the required box-level specific values (Table 1).

Box level	0	1	2	3	4	5	6
N_{box}	1	8	16	64	336	1,546	4,414
l_{box}	3.09	1.54	0.77	0.39	0.19	0.10	0.05
D_{box}	5.35	2.68	1.34	0.67	0.33	0.17	0.08
kD_{box}	224.2	112.1	57.1	28.0	14.0	7.0	3.5
$O_{mp,min}$			67	36	19	11	7
$O_{mp,max}$			147	78	44	25	16
N_{us}			43,808	12,482	3,872	1,250	512

Table 1: Box level specific values resulting from the MLFMM cluster boxing for the cylinder as shown in Fig. 2

with N_{box} number of cubic boxes
l_{box} length of box edge in [m]
D_{box} diameter (diagonal) of box in [m]
kD_{box} $k \cdot D_{box}$ value of box
$O_{mp,min}$ minimum required multipole order for a box
$O_{mp,max}$ maximum required multipole order for a box
N_{us} number of vectors of the unit sphere of a box

The result using the conventional BEM with a GMRES solver is shown in Figure 5 (used as a reference solution).
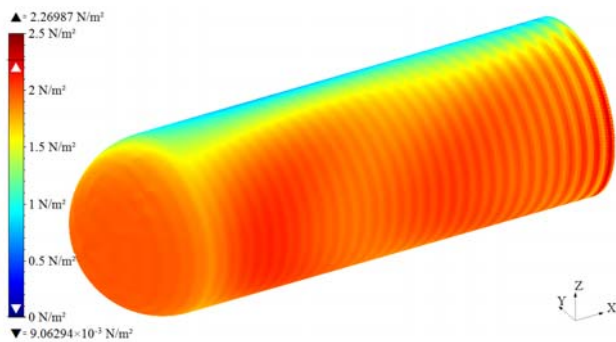

Figure 5: Surface pressure $\mathrm{abs}\left(p_{surf}\right)$ of the cylinder, using the conventional BEM with a GMRES solver (10 kHz, Δt_{solve} = 2,545 s, N_{iter} = 31, $e_{iter,max}$ = 10[-7])

The result of the calculation using the MLFMM with these values is shown in Figure 6. The error within the quality and the quantity of the pressure gradient is clearly visible, especially at the boundaries of the cluster boxes.
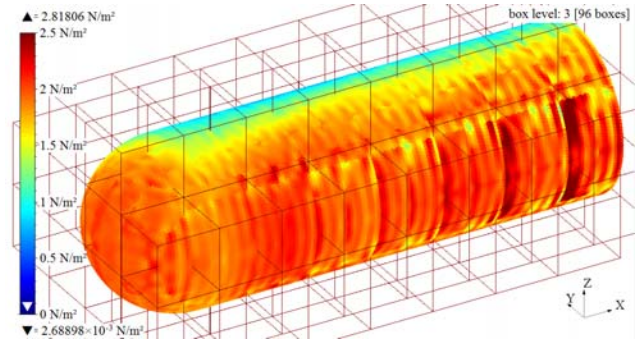

Figure 6: Surface pressure $\mathrm{abs}\left(p_{surf}\right)$ of the cylinder, using the standard MLFMM (with 96 cluster boxes at box level 3, Δt_{solve} = 159 s, N_{iter} = 32, $e_{iter,max}$ = 10[-7])

This is due to the high value of the multipole order ($O_{mp,max}$[2] = 147, see orange field in Table 1), which is required for the translation between clusters at box level 2. The error resulting due to numerical inaccuracies of the Hankel functions on this level later is transformed to the boxes on level 3 and so on.

Source Clustering Method

The idea behind this new method presented is to "remove" the critical M2L-step from the MLFMM algorithm and to calculate the corresponding interactions with a source clustering algorithm. This method is closely related to the so-called Panel Clustering Method, with some differences due to the integration within the MLFMM.

The required cluster interactions on the same box level are defined by the so-called Interaction List 2 ($\mathbb{I}_{l2}$, green boxes), resulting from the MLFMM boxing algorithm (Figure 7).

At first, a conventional BEM Helmholtz integration over all source points (green dots) is used to build a set of SCM coefficients for a "representative" load point in the destination cluster's (blue box) weight center (center of all elements, green circle).
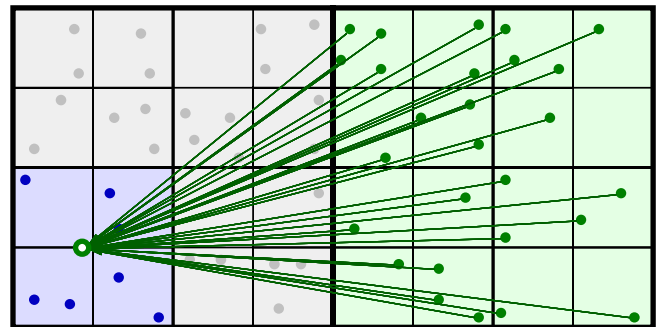

Figure 7: Step 1 of the simple Source Clustering Method, calculation of box center representatives at box level 2

In a second step, a corrective transfer function is used to obtain an approximated part of the "representative" value at each destination point (blue dots) within the current destination cluster (Figure 8, next page). All near-field interactions between the destination points and the source points in the gray boxes are calculated by the MLFMM.
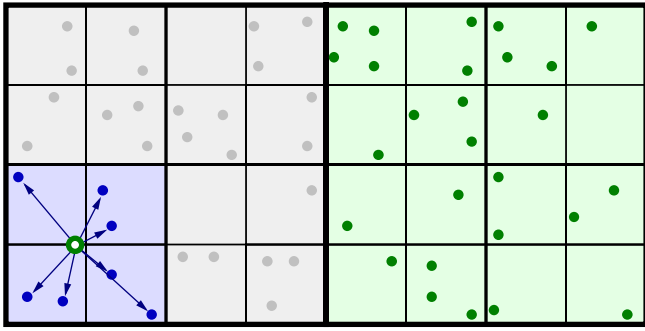
Figure 8: Step 2 of the simple Source Clustering Method, translation to the destination points at box level 2

Results of the SCM (simple version)

The simple version of the SCM only calculates the interactions at box level 2. The result of this combination is better with respect to quantity but the quality is not really satisfactory (Figure 9).
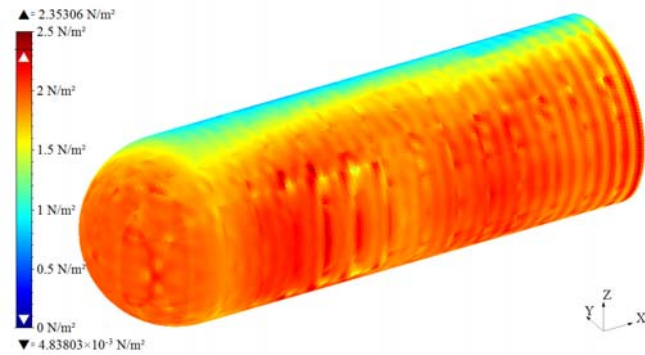

Figure 9: Surface pressure $\text{abs}(p_{surf})$ using the simple SCM algorithm for box level 2 [SCM 3/1] (10 kHz, Δt_{solve} = 95 s, N_{iter} = 31, $e_{iter,max}$ = 10^{-7})

Recursive version of the SCM

In order to obtain better results, a modified recursive version was designed and implemented where the weight centers of the child boxes of the destination cluster are used as the representative points (green circles, Figure 10).
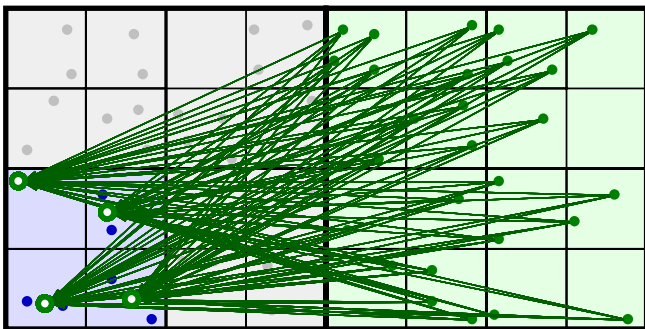

Figure 10: Step 1 of the recursive Source Clustering Method, calculation of representatives at the weight centers of the child boxes of the destination cluster boxes at level 3

The number of the resulting SCM coefficients and the required memory is higher compared to the simple version, but the approximated values at the destination points (Figure 11) are quite more precise and therefore better due to the smaller distances between the representative and the destination points.
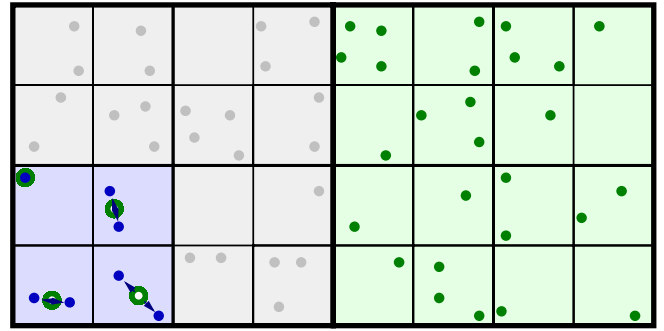

Figure 11: Step 2 of the recursive Source Clustering Method, translation to the destination points at box level 3

Results of the SCM (recursive version)

The result achieved when calculating the M2L-interactions at box levels 2 and 3 with the recursive SCM version is more precise (Figure 12).
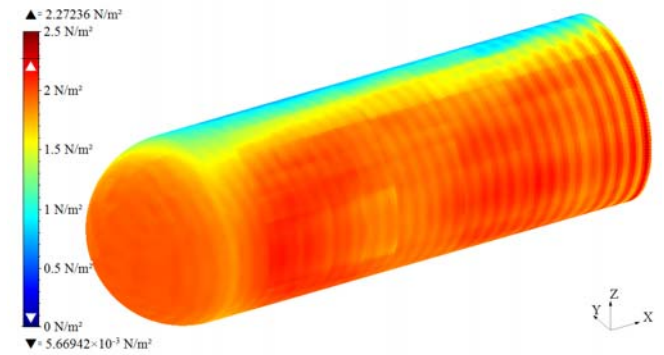

Figure 12: Surface pressure $\text{abs}(p_{surf})$ using the recursive SCM algorithm for box levels 2 and 3 [SCM 4/1] (10 kHz, Δt_{solve} = 71 s, N_{iter} = 31, $e_{iter,max}$ = 10^{-7})

Even if there are still some slight differences with respect to the pressure gradient when comparing with the conventional BEM, the results for the scattered pressure level at a distance of 100 m is nearly exact the same (Figure 13).
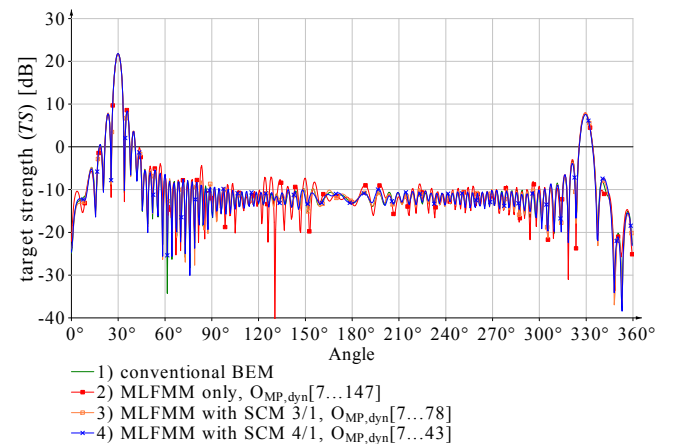

Figure 13: Scattered pressure level $\text{abs}(L_{p,scat})$ on a circle with a distance of 100 m to the center of the structure

Remark: All calculations were done on a Dual XEON workstation with 2,66 GHz, 12 cores and 48 GB RAM, running LINUX and using the GMRES solver of a self-developed parallelized BEM solver application code [6] which supports multiple solvers and different solving methods.

Conclusions

It could be shown that the implemented Source Clustering Method can be used to overcome the problem of the Multipole-to-Local translation step within the Multi-Level Fast Multipole method, when the required multipole order is too high to obtain useable values for the Hankel function.

Another advantage is the lower amount of system memory required for the expansion coefficients of the high-order unit spheres at these levels.

The approximation for the local parts works well when the distance between the source and destination points is big enough to fulfill the "far"-condition of the MLFMM and can be optimized with a recursive version of the presented method.

It is also possible to store the required SCM coefficients when enough memory is available, in so doing the time for the solving process could be reduced in comparison with the standard MLFMM.

Anyway, the number of elements (= source / destination points) is relevant for the SCM solving time and the memory required and so the SCM step can be the most expensive one within the MLFMM matrix-vector-product calculation.

Outlook

Additional work is recommended to optimize the corrective functions and to find the best combination of the box levels calculated by SCM / MLFMM and to optimize an additional sub-leveling step during the recursive part as well.

A first extension to the method using this sub-leveling gives better results (Figure 14) but at higher costs with respect to solution time and memory.
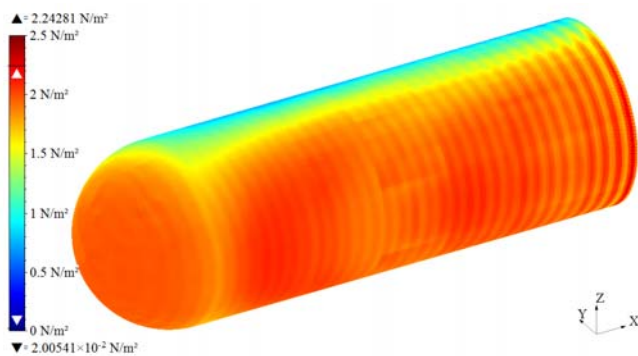

Figure 14: Surface pressure $\mathrm{abs}(p_{surf})$ using the recursive SCM algorithm with sub-leveling for box levels 2 and 3 [SCM 4/2]
(10 kHz, Δt_{solve} = 90 s, N_{iter} = 31, $e_{iter,max}$ = 10^{-7})

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