IMPLEMENTATION AND RESULTS OF A MASS INERTIA COUPLING AS AN EXTENSION OF THE BEM FOR THIN SHELLS

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The conventional 3D boundary element method for the determination of sound radiation from thin shell-like layered structures requires a high computational cost, since a corresponding customized model must be used for discretization, which needs different shell layer areas and leads to a large number of unknowns. For example, the calculation of a coupled problem for one elastic shell requires four discretized boundary element areas.

In many cases, the approximate approach outlined in this paper may be used, which gives acceptable results using a mass inertia coupling for thin structures. The number of required boundary element regions in the example above is reduced to only two, leading to a much smaller order of the system matrix and an accordingly greatly reduced solution time. At this time, the method is suitable primarily within the middle frequency range because it is not able to take into account shear waves.

In this paper the basics of this coupling method are outlined and some results for general coupled structures are given and compared with results obtained from FEM calculations.

1. Introduction

The use of the boundary element method for acoustic calculations taking into account fluid-structure coupling with elastic materials, as presented e.g. in [1] and [2], is complicated and demanding in terms of computing time, because one element of an elastic surface results in nine coefficients in the system of equations resp. matrix.

Similar studies showed significant differences between BE and FE-based calculations and analytical results, especially when using thin elastic layers [3].

1.1 A multiple coupled case (fluid - elastic shell - fluid)

In this example, three areas exist (outer space Ω_0, elastic shell Ω_1 and interior fluid filling Ω_2. Fig. 1), which leads to two coupled edge regions (Γ_1 and Γ_2).

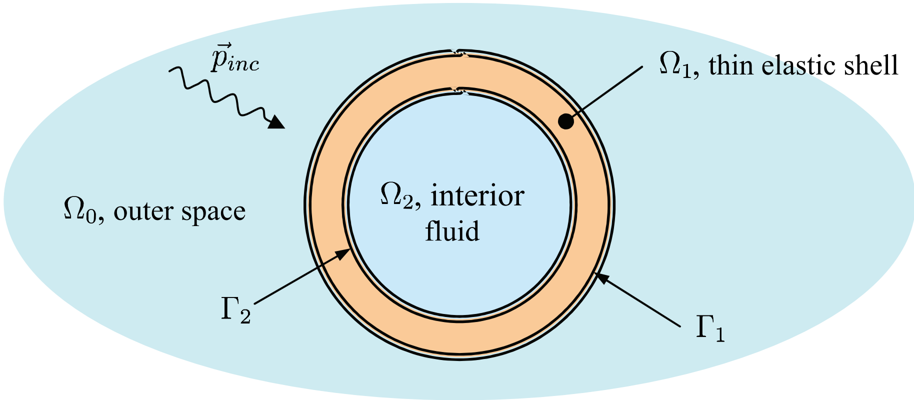

Figure 1. structural setup of a multiple coupled case
(fluid areas: blue, solid areas: brown, based on [3])

The order of the resulting matrix is given by $O_{mx} = N_{f,\Gamma_1} + 3N_{s,\Gamma_1} + 3N_{s,\Gamma_2} + N_{f,\Gamma_2}$ with N_Γ as the number of elements of the respective surface.

Fig. 2 gives the layout of the matrix for such a problem, i.e. an air-filled elastic spherical shell made of a metal in water. Here, the orders of magnitude are chosen such that the number of elements per coupling region is equal ($N_{f,\Gamma_1} = N_{s,\Gamma_1} = N_{s,\Gamma_2} = N_{f,\Gamma_2}$).

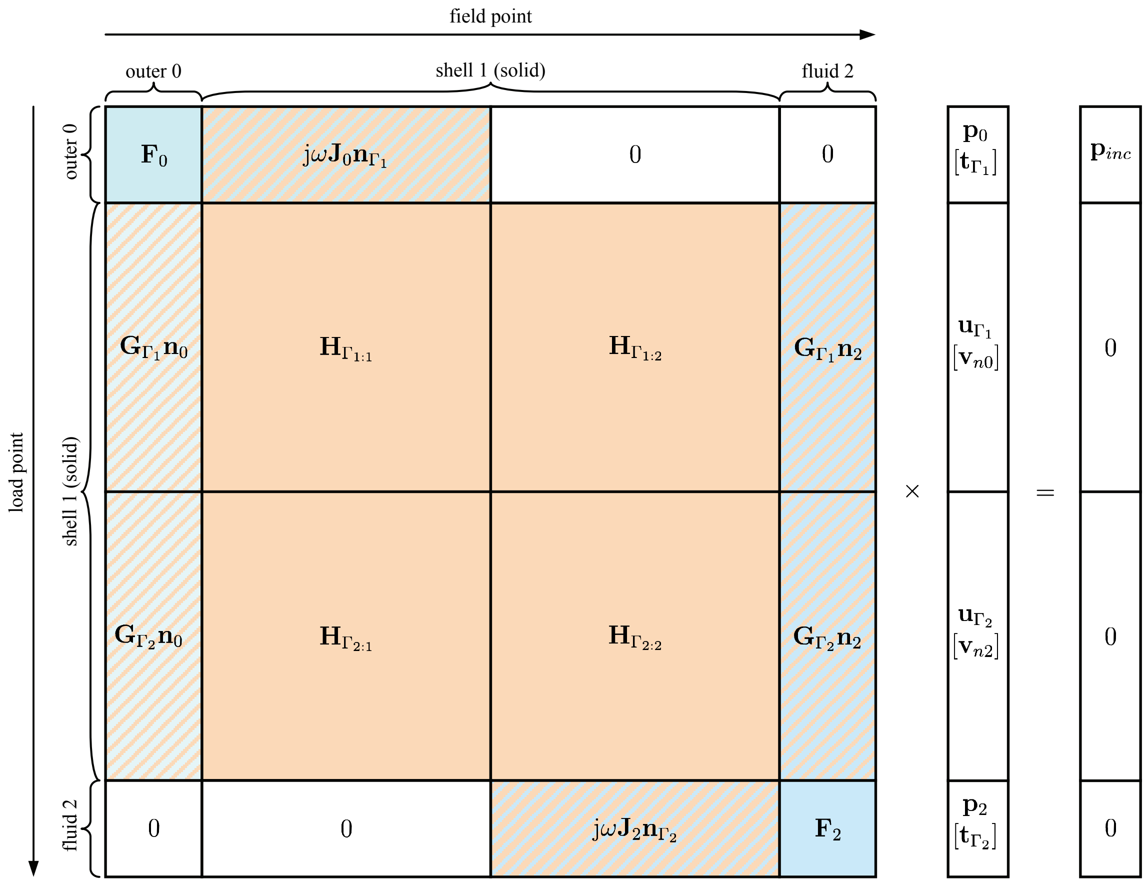

Figure 2. matrix setup of a multiple coupled case
(fluid coefficients: blue, solid coefficients: brown, based on [3])

with

$\mathbf{p}$	pressure
$\mathbf{p}_{inc}$	incident pressure
$\mathbf{v}_n$	normal velocity
$\mathbf{t}$	tension
$\mathbf{u}$	displacement
$\mathbf{F}$, $\mathbf{J}$	matrix coefficients of the corresponding fundamental solution for the fluid material(s)
$\mathbf{G}$, $\mathbf{H}$	matrix coefficients of the corresponding fundamental solution for the elastic material(s) (see [3], pp. 6ff, for the details and formulas of the fundamental solutions)

Replacing the elastic shell with a kind of mass inertia coupling, assuming equal pressure on both sides of one shell element, "removes" the elastic shell region from the structural model (Fig. 3) and results in an impedance equivalent coefficient (shown as $\mathbf{F}_{z,0}$ in Fig. 4) given by the predefined shell width and material density ("shell boundary condition"). This reduces the numbers of unknowns significantly, i.e. to $O_{mx} = 2 \times N_{f,\Gamma}$ for the example shown before.

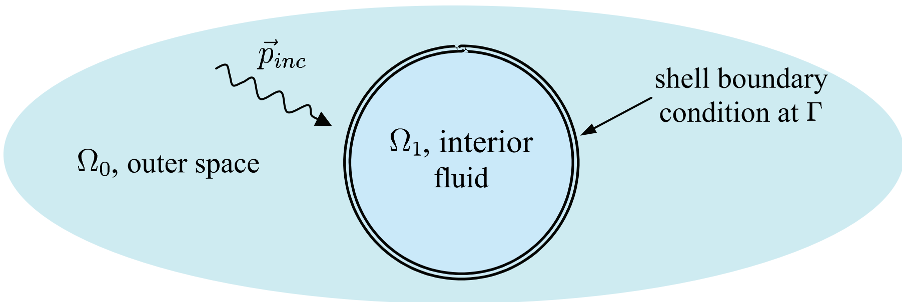

Figure 3. structural setup of a coupled case with shell boundary condition (mass inertia coupling)

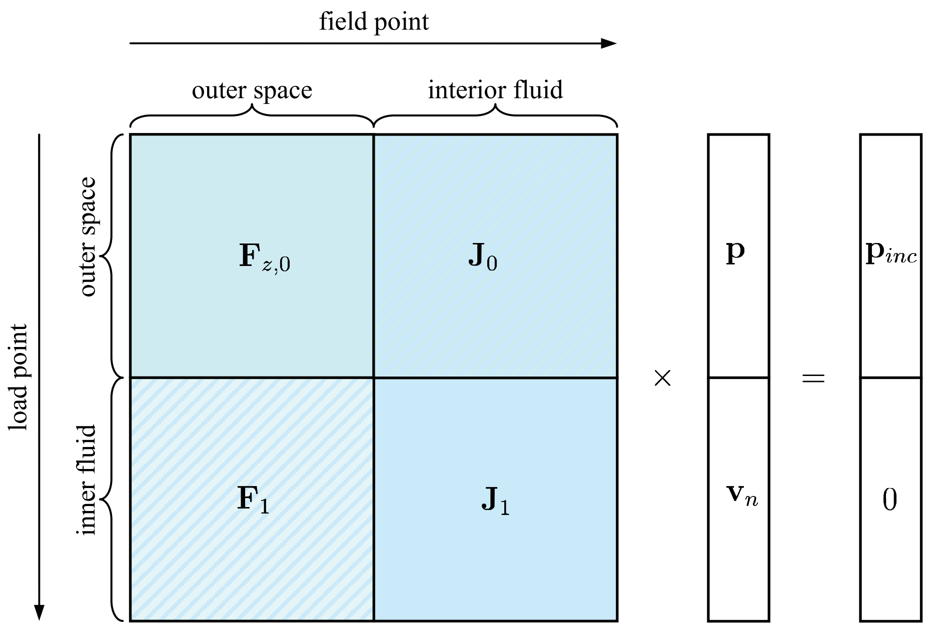

Figure 4. matrix setup of a coupled case with shell boundary condition (mass inertia coupling)

2. Examples and Results

2.1 Closed half spherical shell (bistatic case, single frequency)

Here, a closed half spherical shell with a diameter and total length of r_{sphere} = 0.5 m is used. The shell is made of steel (ρ = 7,850 kg/m³) with a thickness of 1 cm, filled and surrounded by water (ρ = 1,000 kg/m³, c = 1,500 m/s). The 3D model for the coupled BEM consists of 3,380 triangular elements (Fig. 5), this leads to a matrix order of N_{mx} = 7,760. The 2D model used for the rotation-symmetric FEM (COMSOL, Fig. 6, shell area marked in red) consists of about 6,060 elements, including the required PMLs.

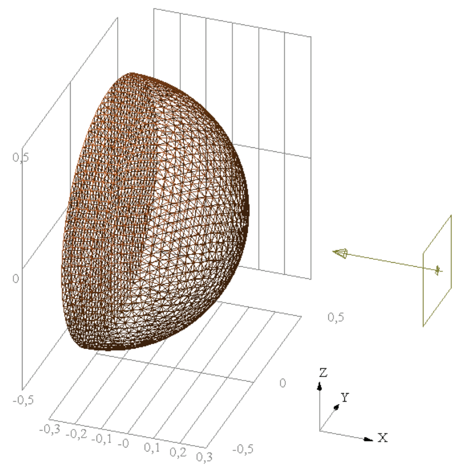

Figure 5. 3D model

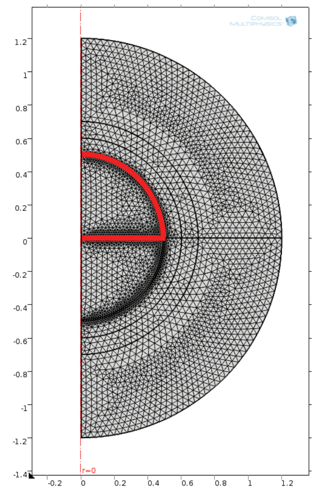

Figure 6. 2D model (red line = shell area)

A plane wave hits the spherical part of the structure (in negative x-direction resp. z-direction in 2D). The results of the normalized pressure level $L_{p,n}$ for a frequency f = 5 kHz are shown in Fig. 7. Results of the FEM were calculated using both a “fluid” steel (without taking into account the shear wave) and an “elastic” steel for comparison.

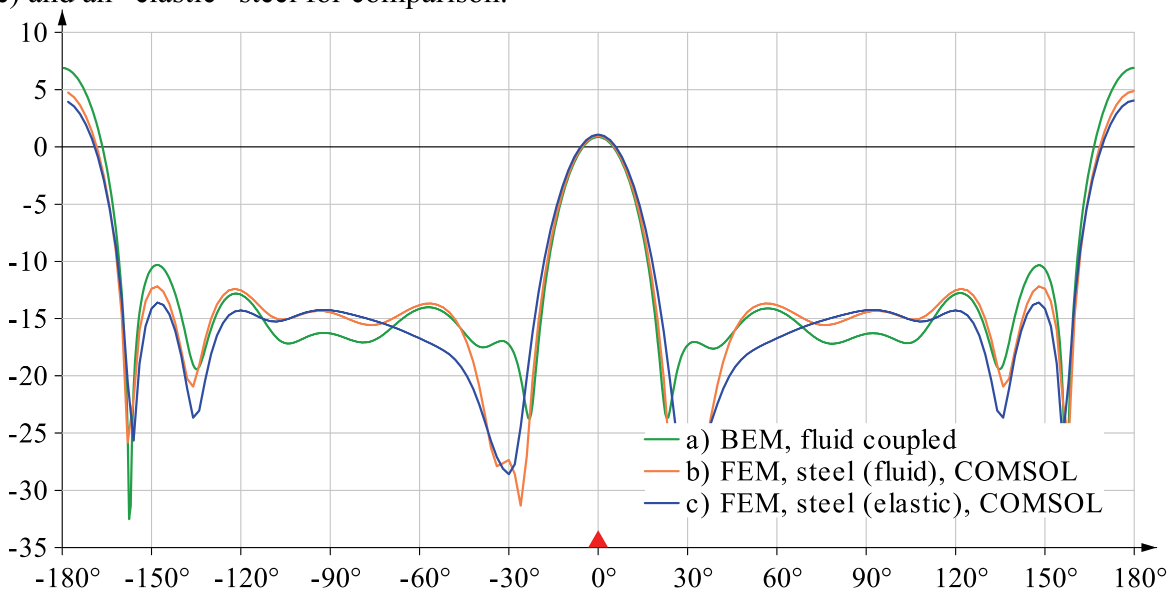

Figure 7. Normalized pressure level $L_{p,n}$ in dB, closed half spherical shell, f = 5 kHz, bistatic case

The results between the “fluid” solutions agree well within a ±2 db range, especially at the monostatic evaluation point at 0° (▲), but in other regions there are some differences when compared with the elastic solution due to the fact, that the shear waves within the elastic material are not taken into account by this BEM coupling approach.

Especially at lower frequencies, these differences are getting bigger, i.e. of approx. 15 dB as shown in Fig. 8 for a frequency of f = 1 kHz at the monostatic evaluation point (0°, ▲, dotted area).

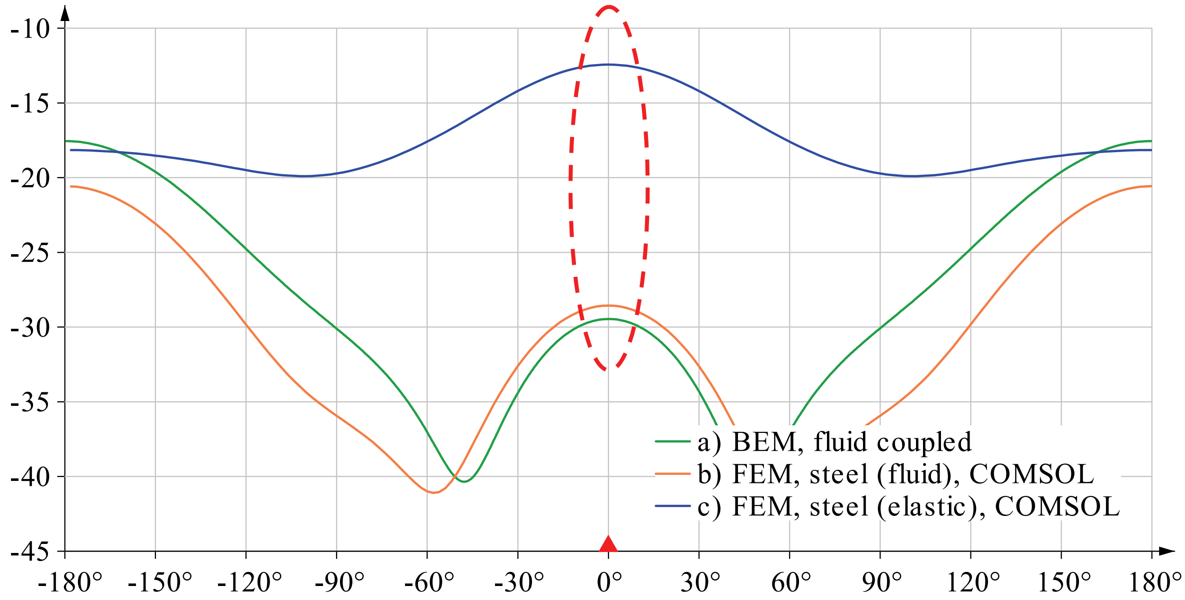

Figure 8. Normalized pressure level $L_{p,n}$ in dB, closed half spherical shell, f = 1 kHz, bistatic case

2.2 Closed half spherical shell (monostatic evaluation, frequency sweep)

These differences can also be clearly seen when one considers a frequency sweep between f = 0.5 and 8 kHz as shown in Fig. 9. The result for the elastic FEM shows the significant peaks due to resonances in the low and middle frequency range and also the strong variances of the sound pressure level at light frequency changes. The result for f = 1 kHz of Fig. 8 can be found at the marked position (▲).

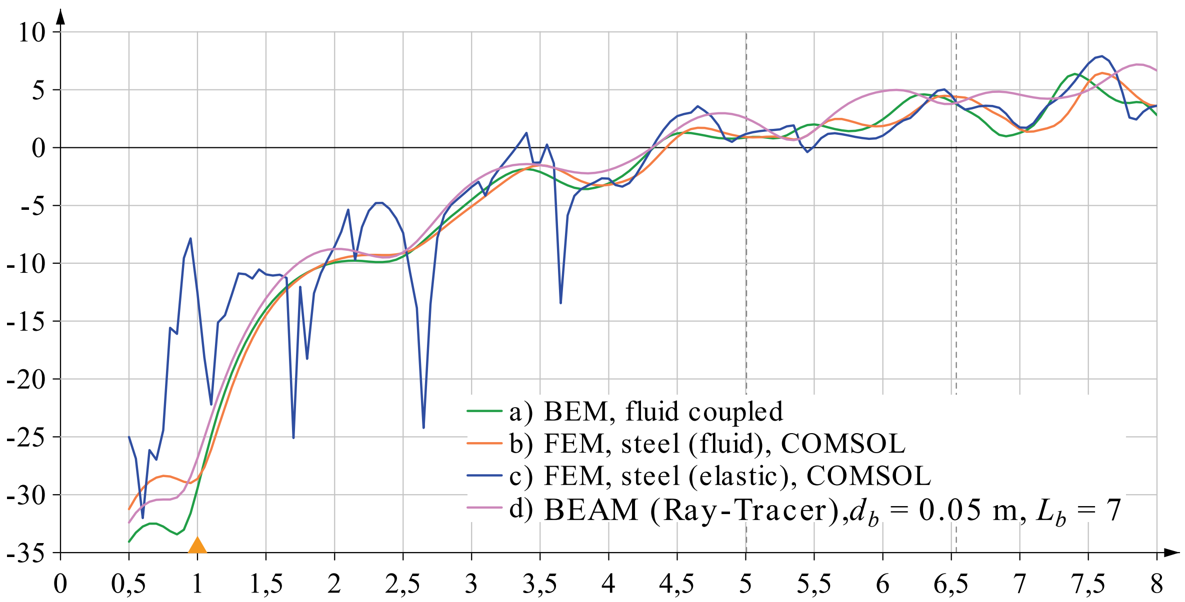

Figure 9. Normalized pressure level $L_{p,n}$ in dB, f = 0.5 ... 8 kHz, monostatic evaluation

The thin vertical dotted lines at 5 and 6.5 kHz are representing the frequency limit of the BEM model used (due to the size of the elements). Additionally, a result of our raytracing-based solver BEAM [4] was added to show its good accordance with the "fluid" solutions.

2.3 Large object with triple mirrors ("BeTSSi model 2")

Another reason for the development of the described BEM coupling method was the desire to make a comparison with results of the BeTSSi II workshop (Benchmark on Target Strength Simulation, September 2014, for details see [6]) and the verification of solutions obtained from the ray-tracing based solver BEAM [4]. One of the models used was a large cylindrical structure ("model 2", Fig. 10). The main body is surrounded by water and filled with air ("air-backed"), including the triple mirror at the right end. Additionally, there are four triple mirrors within a cross-section at the left end, made of 20 mm steel ("water" backed).

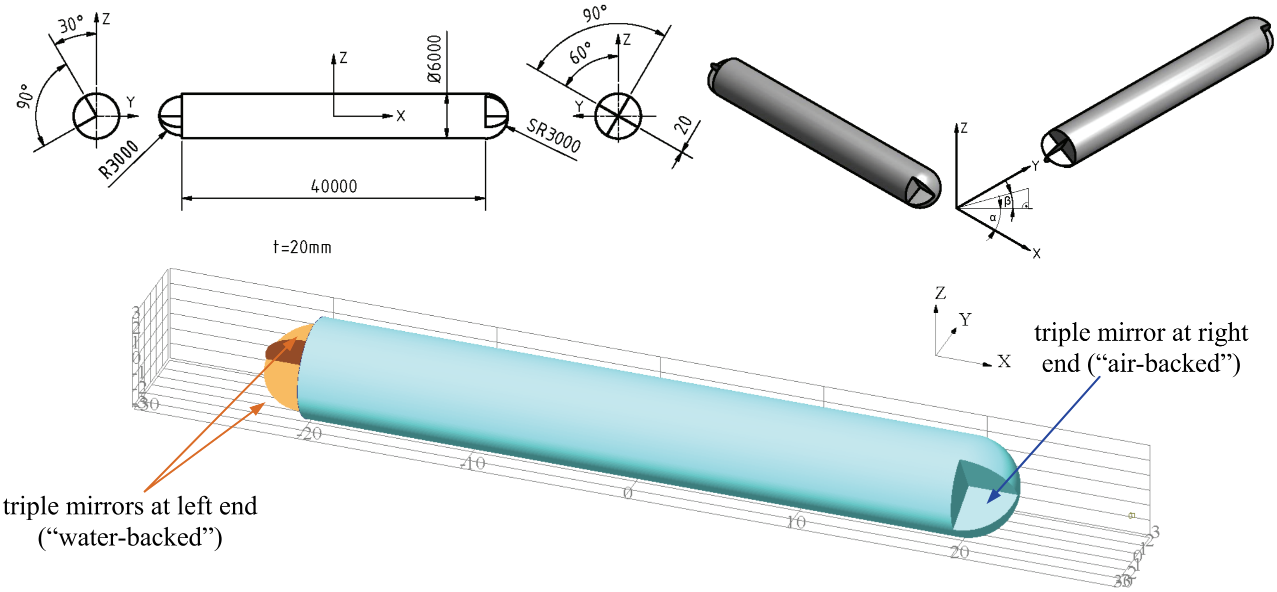

Figure 10. large cylindrical structure with triple mirrors (BeTSSi model 2)

Due to the dimension of this structure and the available computation power, a model consisting of about 217,000 triangular elements was generated to allow a coupled computation using a frequency of $f = 3$ kHz, leading to a matrix order of $O_{mx} = 237{,}906$ and a memory requirement of $O_{mx} \approx 432$ GB (single precision).

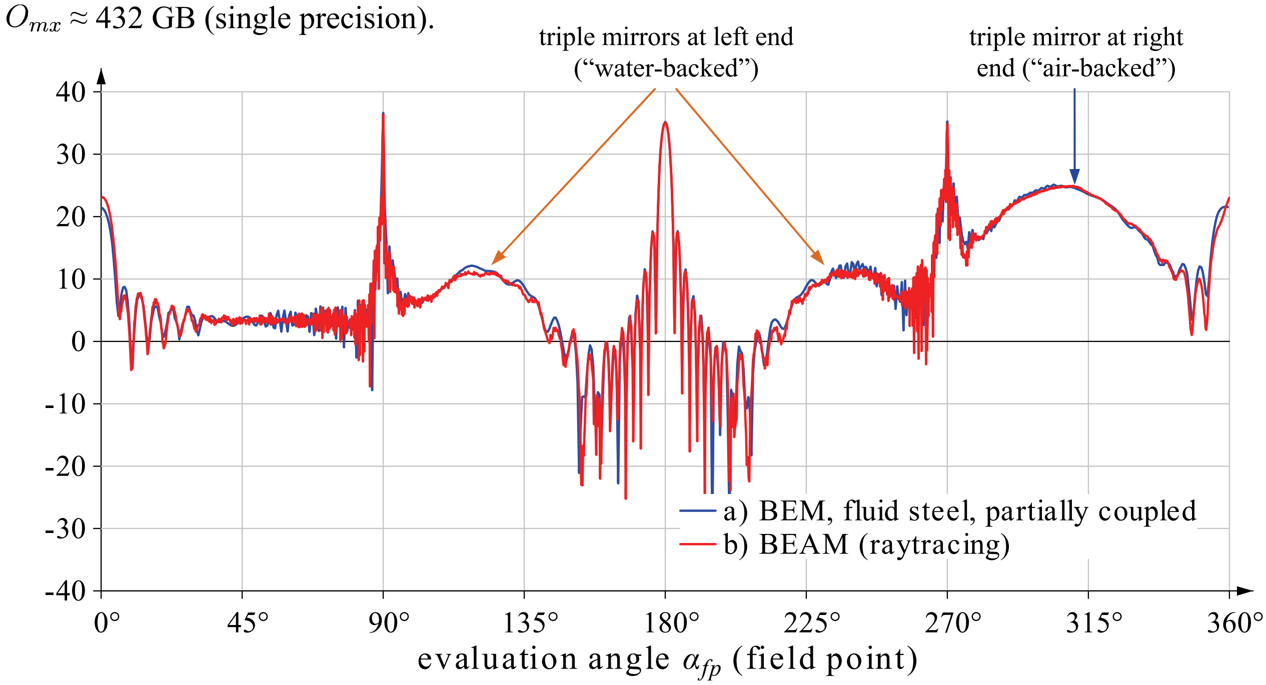

Figure 11. Normalized pressure level $L_{p,n}$ in dB, $f = 3$ kHz, monostatic evaluation

Fig. 11 shows the results for a monostatic evaluation within a 360° circle in the XY plane. Both solutions agree very well within a range of ± 2 dB and the expected differences of the normalized pressure level at the “air-backed” right triple mirror (blue arrow) and the “water-backed” left mirrors (brown arrows) are clearly visible.

The main difference was the computation time of 39,840 s (≈ 11 h) for the BEM solution and 216 s for the ray tracing solver.

3. Conclusions and future work

The results show that the presented approximate BEM coupling method is suitable for the medium frequency range and is bounded for higher frequencies only by the available memory resp. computational power. The quality of the results depends on the size of the considered structure and the wavelength used. The comparison with the results of the BEAM method shows a good agreement for thin shell structures.

In the lower frequency range, the quality of the results is less accurate because the influence of the shear wave cannot currently be taken into account and therefore, the resonances occurring usually cannot be detected.

It should also be investigated whether an approximate consideration of the shear wave is possible through a kind of FEM-based version within the calculation of the coupled elements.

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